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Poisson sigma model over group manifolds

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Abstract

We study the Poisson sigma model which can be viewed as a topological string theory. Mainly we concentrate our attention on the Poisson sigma model over a group manifold $\mathcal{G}$ with a Poisson–Lie structure. In this case the flat connection conditions arise naturally. The boundary conditions (D-branes) are studied in this model. It turns out that the D-branes are labelled by the coisotropic subgroups of $\mathcal{G}$. We give a description of the moduli space of classical solutions over Riemann surfaces both without and with boundaries. Finally we comment briefly on the duality properties of the model.

Keywords: Poisson sigma model; Poisson–Lie structure; D-branes

1. Introduction

The Poisson sigma model introduced in [16,25] is a topological two-dimensional field theory with the tangent space $\mathcal{M}$ being a Poisson manifold. The model is closely related to other two-dimensional models such as gravity models, the Wess–Zumino–Witten models and two-dimensional Yang–Mills theory. Recently the Poisson sigma model has attracted considerable attention due to its relation to deformation quantization. Namely it has been shown in [8] that the perturbative path integral expansion of the Poisson sigma model over the disk leads to the Kontsevich's star product [21].

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In the present paper, we conduct a systematic investigation of the classical Poisson sigma model with the target space being a Poisson–Lie group. We consider the model defined over Riemann surfaces both with and without boundaries. The Poisson–Lie groups are the semiclassical limit of quantum groups, therefore by exploring these models at the quantum level we may hope to find new insights into quantum groups. This could be considered as the main motivation for the project. In the present paper, we take the first step in this direction and explore the classical theory, we hope to come back to the quantum theory elsewhere.

The key observation of the paper is that the Poisson action of a Poisson–Lie group on a target manifold implies the existence of a flat connection in the corresponding model. In particular if the target manifold is a Poisson–Lie group then the on-shell Poisson sigma model can be reformulated in terms of the flat connections of an appropriate principal bundle and the parallel section of an associated fiber bundle. Moreover the infinitesimal on-shell gauge transformations can be interpreted as dressing transformations and integrated to define finite gauge transformations. This allows us to define the space of solutions modulo gauge transformations. Since the dressing transformations are transitive on symplectic leaves, the moduli space can be characterized in terms of the space of leaves. Another important point is that the boundary conditions are labelled by the coisotropic subgroups of the Poisson–Lie group.

Some of these issues have been already addressed in the literature. Previously the Poisson sigma model over the Poisson–Lie group has been considered in [2,13] in connection to $\mathcal{G}/\mathcal{G}$ Wess–Zumino–Witten theories. While our project was in progress the work [7] has appeared where the systematic study of Poisson sigma models over Poisson–Lie groups has been attempted. Despite some intersections between the results of the work [7] and the present paper, hopefully we can offer a reasonably complete picture of the classical model and clarify some important issues. The spaces of classical solutions of the Poisson sigma models have also been discussed previously (e.g., see the recent work [5] and the references therein). The important recent work [9] should be mentioned where the first systematic study of general boundary conditions for the Poisson sigma model has been undertaken. In the present paper, we clarify some general issues and as well as we give an illustration of the possible boundary conditions which are specific for the Poisson–Lie case.

The paper is organized as follows. In Section 2 we review the relevant notions from the theory of Poisson manifolds and Poisson–Lie groups. In Section 3 we recall the definition of the Poisson sigma model and go on to discuss the general boundary conditions for the model in particular. We arrive at the same result as in [9], however the derivation is somewhat different. In Section 4 we analyze the relation between the group action on the target space and the symmetries (and their generalizations) of the Poisson sigma model. The main observation is that the Poisson action of a Poisson–Lie group implies the flat connection conditions for the Poisson sigma model. Then in Section 5 we apply these results to the specific case when the target space is a group manifold itself. The on-shell model can be rewritten in terms of new variables which have a clear geometrical interpretation: the flat connection of the principal bundle and the parallel section of the associated fiber bundle. We also offer the appropriate description of the boundary conditions in this context. Using these results in Section 6 we construct the moduli spaces of the classical solutions of the model over a generic Riemann surface both with and without boundaries. The description that we obtain connects the moduli space to the space of symplectic leaves. This space of leaves describes a very intrinsic property of the Poisson structure. Since our considerations