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Chaos, Solitons and Fractals 25 (2005) 277–286
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Topological reduction of information systems

E.F. Lashin *, T. Medhat

Physics and Engineering Mathematics Department, Faculty of Engineering, Tanta University, 31521, Egypt

Accepted 15 September 2004

Abstract

The main task of the present work is to apply some methods for knowledge reductions in the case of knowledge based on equivalence relations. Topological techniques are applied to construct knowledge bases via general relations. Topological structures are used to obtain discernibility matrix and discernibility function for knowledge reduction and decision making. Topology plays a significant role in quantum physics, high energy physics, and superstring theory. Reduction of attributes can be applied in the process of compactification of space time dimensions.

1. Introduction

Information play an important role in our life. The need for discovering knowledge from information increases with the forward rapid development of the recent civilization. El-Naschie [6] pointed to the uncertainty of information in quantum space-time. Reduction of information via the suggested method was applied in [1]. The process of knowledge discovery takes place along mathematical treatments, among them is the reduction of information systems. One of the most useful methods for this process is the rough set theory approach which depends on partitioning the universe of objects via equivalence relations. The works due to The Wheeler about importance of information analysis in physics can be found in [5]. Topologically we can say the mathematical treatments depend on a special class of topological spaces in which every open set is closed, in this class of spaces many recent topological class of subsets concedes with the class of open and closed sets and this in turn limits the application. The purpose of this work is to generalize the tool to a general binary relation in the place of the equivalence relations. We establish topological structures associated with this general relations and introduce a view for obtaining discernibility matrix and a method for knowledge reduction. However our approach is the same as Pawlak's one [10,11] if the relation is an equivalence relation. The proposed topological structure we give have will open the way for using new topological results in the process of knowledge discovery for information systems whose basic classes are not pairwise disjoint. In this paper, we discuss the effect of information reduction and specialize of information classification (resolution) on the accuracy of approximation. In physics, this appear clearly for example see [9, pp. 8–9] . Relations of topology and physics have been appeared in [7,8,12].

* Corresponding author.

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.09.107

2. Basic concepts

2.1. Knowledge base

The approximation space [10,11] is a pair of (U, R), where U is a non-empty finite set of objects (states, patients, digits, cars, ...etc) called a universe and R is an equivalence relation over U which makes a partition for U, i.e. a family $C = \{X_1, X_2, X_3, \ldots, X_n\}$ such that $X_i \subseteq U$, $X_i \neq \phi$, $X_i \cap X_j = \phi$ for $i \neq j$, $i,j = 1,2,3,\ldots,n$ and $\cup X_i = U$, the class C is called the knowledge base of (U, R).

The universe U of objects with relation R play an important role in converting data into knowledge which use R as a tool of a mathematical model for dealing with members and subsets of U. Thus we can say that R changes U from just being a set to a mathematical model.

The following is an example of an approximation space and its associated knowledge base.

Example 2.1. Let $U = \{1,2,3,4,5,6,7,8,9\}$ and relation R which partition U into two classes even and odd numbers, then knowledge base is:

$$U/R = \{\{1,3,5,7,9\},\{2,4,6,8\}\}, \quad X_1 = \{1,3,5,7,9\}, \quad X_2 = \{2,4,6,8\}, \quad X_1 \cap X_2 = \phi, \quad X_1 \cup X_2 = U$$

2.2. Information systems

An information system S [3,4] can be defined as $S = (U, A, p, V)$, where U is a non-empty finite set of objects called a universe and A is a non-empty finite set of attributes (features, variables, characteristic conditions, leds, ...etc).

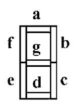
$$S = (U, A, p, V)$$

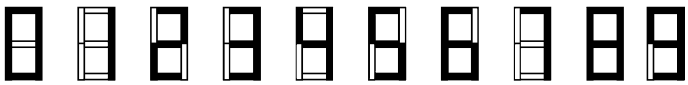
Any subset $X \subseteq U$ wil be called a concept or a category in U, Each attribute $a \in A$ can be viewed as a function that maps elements of U into a set Va, where the set Va is called the value of set of the attribute a.

$$p : UxA \to Va$$

In the following, we reformulated an example for information system using example given in [10].

Example 2.2. The seven segment display which gives us the numbers from 0 to 9 as shown below:





The structure of each digit is shown in the information system Table 1:

The objects are digits 0,1,2,3...,9 and attributes are leds a,b,c,d,e,f and g of display. We find that $Va = Vb = Vc = Vd = Ve = Vf = Vg = \{0,1\}$ & $1 \equiv$ ON, $0 \equiv$ OFF. and $a(5) = 1$, $b(5) = 0$, $g(9) = 1$.

Table 1

U:A	a	b	c	d	e	f	g
0	1	1	1	1	1	1	0
1	0	1	1	0	0	0	0
2	1	1	0	1	1	0	1
3	1	1	1	1	0	0	1
4	0	1	1	0	0	1	1
5	1	0	1	1	0	1	1
6	1	0	1	1	1	1	1
7	1	1	1	0	0	0	0
8	1	1	1	1	1	1	1
9	1	1	1	1	0	1	1