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Parameter mismatches, variable delay times and synchronization in time-delayed systems

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Abstract

We investigate synchronization between two unidirectionally linearly coupled chaotic non-identical time-delayed systems and show that parameter mismatches are of crucial importance to achieve synchronization. We establish that independent of the relation between the delay time in the coupled systems and the coupling delay time, only retarded synchronization with the coupling delay time is obtained. We show that with parameter mismatch or without it neither complete nor anticipating synchronization occurs. We derive existence and stability conditions for the retarded synchronization manifold. We demonstrate our approach using examples of the Ikeda and Mackey Glass models. Also for the first time we investigate chaos synchronization in time-delayed systems with variable delay time and find both existence and sufficient stability conditions for the retarded synchronization manifold with the coupling-delay lag time. Also for the first time we consider synchronization between two unidirectionally coupled chaotic multi-feedback Ikeda systems and derive existence and stability conditions for the different anticipating, lag, and complete synchronization regimes.

1. Introduction

Seminal papers on chaos synchronization [1] have stimulated a wide range of research activity especially extensively in lasers, electronic circuits, chemical and biological systems [2]. Possible application areas of chaos synchronization are in secure communications, optimization of non-linear system performance, modeling brain activity and pattern recognition phenomena [2].

There are different types of synchronization in interacting chaotic systems. Complete, generalized, phase, lag and anticipating synchronizations of chaotic oscillators have been described theoretically and observed experimentally.

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Complete synchronization implies coincidence of states of interacting systems, $y(t) = x(t)$ [1]. Generalized synchronization is defined as the presence of some functional relation between the states of response and drive, i.e. $y(t) = F(x(t))$ [3]. Phase synchronization means entrainment of phases of chaotic oscillators, $n\Phi_x - m\Phi_y = \mathrm{const}$, ($n$ and m are integers) whereas their amplitudes remain chaotic and uncorrelated [4]. Lag synchronization *for the first time* was introduced by Rosenblum et al. [5] under certain approximations in studying synchronization between *bi-directionally* coupled systems described by the ordinary differential equations (no intrinsic delay terms) with *parameter mismatches*: $y(t) \approx x_\tau(t) \equiv x(t-\tau)$ with positive τ. Anticipating synchronization [6–8] also appears as a coincidence of shifted-in-time states of two coupled systems, but in this case the driven system anticipates the driver, $y(t) = x(t+\tau)$ or $x = y_\tau$, $\tau > 0$. An experimental observation of anticipating synchronization in external cavity laser diodes [9] has been reported recently, see also [10] for the theoretical interpretation of the experimental results. The concept of inverse anticipating synchronization $x = -y_\tau$ is introduced in [11].

Due to finite signal transmission times, switching speeds and memory effects time-delayed systems are ubiquitous in nature, technology and society [12]. Therefore the study of synchronization phenomena in such systems is of high practical importance. Time-delayed systems are also interesting because the dimension of their chaotic dynamics can be made arbitrarily large by increasing their delay time. From this point of view these systems are especially appealing for secure communication schemes [13].

Role of parameter mismatches in synchronization phenomena is quite versatile. In certain cases parameter mismatches are detrimental to the synchronization quality: in the case of small parameter mismatches the synchronization error does not decay to zero with time, but can show small fluctuations about zero or even a non-zero mean value; larger values of parameter mismatches can result in the loss of synchronization [8,14]. In some cases parameter mismatches change the time shift between the synchronized systems [15]. In certain cases their presence is necessary for synchronization. We reiterate that the crucial role of parameter mismatches for lag synchronization between *bi-directionally* coupled systems was first studied in [5] by Rosenblum et al. As such, lag synchronization cannot be observed if two oscillators are completely identical, see e.g. [16] and references therein.

Multi-feedback and multi-delay systems are ubiquitous in nature and technology. Prominent examples can be found in biological and biomedical systems,laser physics, integrated communications [12]. In laser physics such a situation arises in lasers subject to two or more optical or electro-optical feedback. Second optical feedback could be useful to stabilize laser intensity [17]. Chaotic behaviour of laser systems with two optical feedback mechanism is studied in recent works [18]. To the best of our knowledge chaos synchronization between the multi-feedback systems is to be investigated yet. Having in mind enormous application implications of chaos synchronization e.g. in secure communication, investigation of synchronization regimes (lag, complete, anticipating etc.) in multi-feedback systems is of immense importance.

In this paper we investigate synchronization between the two *unidirectionally* coupled chaotic non-identical time-delayed systems having a fairly general form of coupling and show for the first time that parameter mismatches are, in fact, of crucial importance for achieving synchronization. We show that independent of the relation between the delay time in the coupled systems and the coupling delay time, only retarded (lag) synchronization is obtained. (Usually for lag synchronization between the unidirecitionally coupled time-delayed systems the term retarded synchronization is prefered [8].) In this case the lag time is the coupling delay time. We consider both constant and variable feedback delay times. We demonstrate our approach using examples of the Ikeda and Mackey Glass models.

In the paper also for the first time we investigate synchronization between two unidirectionally coupled chaotic multi-feedback Ikeda systems and find both existence and stability conditions for different synchronization regimes.

2. General theory

Consider a situation where a time-delayed chaotic master (driver) system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\alpha_1 x + k_1 f(x_{\tau_1}), \tag{1}$$

drives a non-identical slave (response) system

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\alpha_2 y + k_2 f(y_{\tau_1}) + k_3 x_{\tau_2}, \tag{2}$$

where x and y are dynamical variables; $f(x)$ is differentiable non-linear function; α_1 and α_2 are relaxation coefficients for the driving and driven dynamical variables, respectively:throughout the paper we assume that $\alpha_1 = \alpha - \delta$ and $\alpha_2 = \alpha + \delta$, δ determines the mismatch of relaxation coefficients; τ_1 is the feedback delay time in the coupled systems; τ_2 is the cou-