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Adaptive synchronization of Rossler system with uncertain parameters

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Abstract

This article addresses control for the chaos synchronization of Rossler systems with three uncertain parameters. Based on the Lyapunov stability theory, an adaptive control law is derived to make the states of two identical Rossler systems asymptotically synchronized. A numerical simulations is presented to show the effectiveness of the proposed chaos synchronization scheme.

1. Introduction

Since Pecora and Carroll [1] introduced a method to synchronization two identical chaotic systems with different initial conditions, chaos synchronization have attracted a great deal of attention from various fields and has been studied extensively by many researchers during the last decades [2–19]. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. Many methods and techniques for handling chaos control and synchronization have been developed, such as PC method [1], OGY method [3], feedback approach [8,9], adaptive method [20], time-delay feedback approach [7], and backstepping design technique [14], etc. Recently, the chaos synchronization of linearly coupled chaotic systems is investigated by Li et al. [15], Lü et al. [16], and Park [17]; the synchronization problem via nonlinear control scheme is studied by Chen and Han [11] and Chen [18]. Also, Wang et al. [20] discussed the linear feedback and adaptive control scheme for the synchronization of Chen system via a single variable. Lu et al. [21] extended the problem to a unified chaotic system. However, most of research are based on the exactly knowing of the system parameters. But in real situation, some or all of the parameters are unknown. In this regard, Han et al. [22] and Elabbasy et al. [23] studied the adaptive feedback synchronization of Lü system with uncertain parameters. Also, Yassen [24] investigated the problem for Rossler system.

In this article, the problem of chaos synchronization to Rossler system with three uncertain parameters is considered. For chaotic synchronization of the system, a novel adaptive control scheme has been proposed. Then, the chaos synchronization of the system is proved by the Lyapunov stability theory.

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The organization of this article is as follows. In Section 2, the problem statement and master-slave synchronization scheme are presented for Rossler system. In Section 3, we provide a numerical example to demonstrate the effectiveness of the proposed method. Finally concluding remark is given.

2. Chaos synchronization of Rossler system

Consider the Rossler system described by

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = b + z(x - c), \end{cases} \tag{1}$$

where x, y, z are state variables, and a, b and c are the positive real constants.

Actually, the system is chaotic when $a = b = 0.2$ and $c = 5.7$. We assume that we have two Rossler systems where the master system with the subscript m drives the slave system having identical equations denoted by the subscript s. For the systems (1), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x}_m = -y_m - z_m, \\ \dot{y}_m = x_m + ay_m, \\ \dot{z}_m = b + z_m(x_m - c), \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x}_s = -y_s - z_s - u_1, \\ \dot{y}_s = x_s + a_1 y_s - u_2, \\ \dot{z}_s = b_1 + z_s(x_s - c_1) - u_3, \end{cases} \tag{3}$$

where the lower scripts m and s stand for the master systems, the slave one, respectively, a_1, b_1 and c_1 are parameters of the slave system which needs to be estimated, and u_1, u_2 and u_3 are the nonlinear controller such that two chaotic systems can be synchronized.

Subtracting Eq. (2) from Eq. (3) yields error dynamical system between Eqs. (2) and (3)

$$\begin{cases} \dot{e}_1(t) = -e_2 - e_3 - u_1, \\ \dot{e}_2(t) = e_1 + a_1 y_s - a y_m - u_2, \\ \dot{e}_3(t) = (b_1 - b) + z_s x_s - z_m x_m - z_s c_1 + z_m c - u_3, \end{cases} \tag{4}$$

where

$$\begin{cases} e_1(t) = x_s(t) - x_m(t), \\ e_2(t) = y_s(t) - y_m(t), \\ e_3(t) = z_s(t) - z_m(t). \end{cases} \tag{5}$$

Here, our goal is to make synchronization between two Rossler systems by using adaptive control scheme $u_i, i = 1,2,3$ when the parameter of the drive system is unknown and different with those of the response system, i.e.,

$$\lim_{t\to\infty} \|e(t)\| = 0,$$

where $e = [e_1\ e_2\ e_3]^{\mathrm{T}}$.

For two identical Rossler systems without control ($u_i = 0,\ i = 1,2,3$), if the initial condition $(x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled Rossler systems, the two systems will approach synchronization for any initial condition by appropriate control gain. For this end, we propose the following control law for the system (3):

$$\begin{aligned} u_1 &= e_1 + (z_m - 1)e_3, \\ u_2 &= (1 + a_1)e_2, \\ u_3 &= (1 + x_s - c_1)e_3, \end{aligned} \tag{6}$$