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An approach of parameter estimation for non-synchronous systems

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Abstract

Synchronization-based parameter estimation is simple and effective but only available to synchronous systems. To come over this limitation, we propose a technique that the parameters of an unknown physical process (possibly a non-synchronous system) can be identified from a time series via a minimization procedure based on a synchronization control. The feasibility of this approach is illustrated in several chaotic systems.

Adjacent state vectors governed by two copies of nonlinear dynamical systems may evolve into a state utterly uncorrelated, but could be synchronized if let one (drive system) drive the other (response system) though a coupling. This synchronization is named as identical synchronization [1] that can be utilized to identify parameters of an unknown physical process via an observed time series [2,3].

Assume that the knowledge of the governing equations $\mathbf{F}(\mathbf{x},\mathbf{p})$ of a physical process is available but parameters $\mathbf{p}$ of the physical process are not known. In synchronization-based parameter estimation, a model system $\mathbf{F}(\mathbf{y},\mathbf{q})$ is used as a response system driven by a time series observed from the evolution of a variable of the physical process (drive system). The model system has the same structure as the physical process but the model parameters and initial state can only be guessed. In general, there is mismatch of the parameters and initial state between the two systems, i.e. $\mathbf{p} \neq \mathbf{q}$ and $\mathbf{x}_0 \neq \mathbf{y}_0$. If the two systems possess the synchronization nature that states of the two systems converge together under the conditions $\mathbf{x}_0 \neq \mathbf{y}_0$ and $\mathbf{p} = \mathbf{q}$, then the model parameters can be detected through a minimization procedure. In minimization, an averaged (accumulated) synchronization error between the original and model systems is formulated as an objective function. Since the synchronization error is smoothly dependent on the parameter mismatch [2], a process of minimization of the error by adjusting the model parameters to the original parameters allows one to carry out parameter estimation. Once the model parameters are tuned to the actual ones, synchronization error subsequently tends to zero. A different strategy [3] proposed by Parlitz uses the auto-synchronization nature for parameter estimation. The technique is to derive a set of ordinary differential equations (ODEs) that possess a null solution for the model parameter evolution. Based on this strategy, a method derived from a global Lyapunov function has an advantage of global stability but the methodology is hardly applicable to many chaotic systems. The other method is to construct a

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local gradient map (the ODEs of model parameters) around the values of the original parameters (assumed unknown). The method may be generally applicable with the limitation that the initial values of the model parameters should be guessed close enough to the actual ones so as to make the linear approximation available.

Synchronization-based parameter estimation offers a simple way in detecting the parameters of an unknown physical process. It is because the methods [2,3] use the feature of the automatic convergence of synchronization between two copies of systems. This feature makes a minimization process less sensitive to the mismatch of initial states between the model and original systems such that the parameters can be detected in lower search dimensions. While by some other techniques [4,5], the mismatch of initial states requires additional efforts in minimization and increases the toughness in parameter identification.

We note that synchronization-based approaches are applicable only when the synchronization nature exists between a model system and a physical process. The occurrence of synchronization depends on not only the parameter values of a physical system but also the coupling form of a variable (observable). A simple example is to couple a pair of Lorenz systems through the z variable (if z is an observable). The resulting dynamics involves a projective synchronization [6], not an identical synchronization. Many dynamical (chaotic) systems may not possess synchronization nature in a wide spectrum of parameter settings. If a physical process is non-synchronous system, the parameter estimation by synchronization-based methods [2,3] becomes impossible.

In this paper, we present a method that enables the synchronization-based identification techniques available to any non-synchronous dynamical systems. The idea is in fact simple. We introduce control terms into the model system. The control terms consist of feedback gains multiplying a synchronization error formed from an observable. The role of the control is to stabilize the model system onto a synchronization manifold of the targeted physical system. Once synchronization is created, the synchronization-based technique [2] becomes effective in a course of identification of system parameters.

We consider two n-dimensional (physical and model) systems coupled with an observable $s \in \mathbf{x}$, described by a set of ordinary differential equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{F}(\mathbf{x}, s, \mathbf{p}) \\ \dot{\mathbf{y}} &= \mathbf{F}(\mathbf{y}, s, \mathbf{q}) \end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbf{R}^n$ denotes the state vector of an unknown physical process, $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T \in \mathbf{R}^n$ the state vector of a model system. The function vector $\mathbf{F} = (f_1, \ldots, f_n)^T$ is known. The original parameters $\mathbf{p} = (p_1, p_2, \ldots, p_m) \in \mathbf{R}^m$ of the physical process are unknown. The model parameters $\mathbf{q} = (q_1, q_2, \ldots, q_m) \in \mathbf{R}^m$ are to be detected. The signal s is a time series observed from a variable of $\mathbf{x}$ of the physical (drive) system. We assume that synchronization over all the variables between the drive and response systems (1) is not available at $\mathbf{p} = \mathbf{q}$.

Consider a specific example of the non-synchronous Lorenz system with the setting

$$\begin{aligned} \dot{x}_1 &= p_1(x_2 - x_1) \\ \dot{x}_2 &= (p_2 - x_3)x_1 + p_3 x_2 \\ \dot{x}_3 &= x_1 x_2 - p_4 x_3 \end{aligned} \tag{2}$$

where the original parameters are $\mathbf{p} = (p_1, p_2, p_3, p_4) = (10, 60, -1, 8/3)$. Let $s = x_1$ be the observable (experimental data) driving a model system by replacing y_1 (i.e., $s = x_1 = y_1$) in the first equation of the model system

$$\begin{aligned} \dot{y}_1 &= q_1(y_2 - s) \\ \dot{y}_2 &= (q_2 - y_3)y_1 + q_3 y_2 \\ \dot{y}_3 &= y_1 y_2 - q_4 y_3 \end{aligned} \tag{3}$$

Even if we set $\mathbf{q} = \mathbf{p}$, synchronization between (2) and (3) is not possible. To create synchronization, we introduce a controller into the model system (3). The controller is assigned as $k(s - y_1)$ where k is a control gain and $(s - y_1)$ is a synchronization error. The controlled model system is thus given by

$$\begin{aligned} \dot{y}_1 &= q_1(y_2 - s) + k(s - y_1) \\ \dot{y}_2 &= (q_2 - y_3)y_1 + q_3 y_2 \\ \dot{y}_3 &= y_1 y_2 - q_4 y_3 \end{aligned} \tag{4}$$

When we adjust the control gain to $k = 29$, synchronization happens. Based on this controlled model system, we can use a minimization technique for parameter estimation. Once the parameters are identified as $\mathbf{p} = \mathbf{q}$, synchronization error tends to zero, i.e. $\mathbf{x} = \mathbf{y}$ for $t \to \infty$. As a result, the control term $k(s - y_1)$ disappears and the controlled model system (4) becomes identical to the original system (2). This example shows a possible way that parameter estimation of non-synchronous systems can be carried out by means of a synchronization control.