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Bäcklund transformation and soliton solutions for KP equation

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Abstract

A new representation of N soliton solution and novel N soliton solution for the KP equation are derived through a new form Bäcklund transformation.

1. Introduction

The KP equation

$$u_t = u_{xxx} + 6uu_x + 3\partial^{-1}u_{yy} \tag{1.1}$$

was first introduced by Kadomtsev and Peteviashvili [1] in order to study the stability of one-dimensional soliton against transverse perturbations. The N soliton solution for the KP equation was obtained by various methods, for instance, the inverse scattering method [2], Hirota method [3], bilinear Bäcklund transformation [4], the trace method [5,6] and Wronskian technique [7] et al. The novel N soliton solution for the KP equation was derived by use of Hirota method [8].

Recently, Zhang and Chen [9,10] have obtained a modified BT by a dependent transformation for some soliton equations, from which some novel soliton can be derived through the Hirota method. In this paper, we would like to consider the solutions of the KP equation similar to Ref. [9,10]. First we present a new form BT in bilinear form through a transformation. Then a new representation of N soliton solution and novel N soliton solution can been derived from the Hirota expansion for special choices of parameter, where the novel N soliton solution was not obtained in Ref. [9,10].

The paper is organized as following. In Section 2, we write the BT in a new bilinear form. In Section 3, the exact solutions for the KP equation are derived by the new form bilinear BT. Finally, a conclusion is given.

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2. New form Bäcklund transformation for the KP equation

The bilinear BT [4] for the KP Eq. (1.1) is

$$D_y g\cdot f = D_x^2 g\cdot f, \tag{2.1a}$$

$$D_t g\cdot f = (D_x^3 + 3D_xD_y)g\cdot f, \tag{2.1b}$$

where D is the well-known operator defined by

$$D_t^m D_x^n a\cdot b = (\partial_t - \partial_{t'})^m(\partial_x - \partial_{x'})^n a(t,x)b(t',x')|_{t'=t,x'=x}. \tag{2.2}$$

The soliton solutions for the KP equation can be denoted by [11]

$$u = 2(\ln f)_{xx}. \tag{2.3}$$

Replacing f by $e^{\xi}f$ and g by $e^{\eta}g$ in Eq. (2.1), according to the formula

$$D_x^m D_y^l e^{\xi}f\cdot e^{\eta}g = e^{\xi+\eta}[D_x + (k-h)]^m[D_y + (p-q)]^l f\cdot g, \tag{2.4a}$$

$$D_t^n e^{\xi}f\cdot e^{\eta}g = e^{\xi+\eta}[D_t + (\omega-\sigma)]^n f\cdot g, \tag{2.4b}$$

$$\xi = kx + wt + py + \xi^{(0)}, \qquad \eta = hx + \sigma t + qy + \eta^{(0)}, \tag{2.4c}$$

we can get the new form bilinear BT

$$D_y g\cdot f - D_x^2 g\cdot f - 2KD_x g\cdot f = 0, \tag{2.5a}$$

$$D_t g\cdot f - D_x^3 g\cdot f - 3D_xD_y g\cdot f - 6KD_x^2 g\cdot f - 12K^2 D_x g\cdot f = 0, \tag{2.5b}$$

where K is a new parameter. Expanding f and g as

$$f = 1 + f^{(1)}\epsilon + f^{(2)}\epsilon^2 + f^{(3)}\epsilon^3 + \cdots, \tag{2.6a}$$

$$g = 1 + g^{(1)}\epsilon + g^{(2)}\epsilon^2 + g^{(3)}\epsilon^3 + \cdots. \tag{2.6b}$$

Substituting Eq. (2.6) into (2.5) and equating coefficients of ϵ yield

$$g_y^{(1)} - f_y^{(1)} - g_{xx}^{(1)} - f_{xx}^{(1)} - 2K(g_x^{(1)} - f_x^{(1)}) = 0, \tag{2.7a}$$

$$g_y^{(2)} - f_y^{(2)} - g_{xx}^{(2)} - f_{xx}^{(2)} - 2K(g_x^{(2)} - f_x^{(2)}) = -D_y g^{(1)}\cdot f^{(1)} + D_x^2 g^{(1)}\cdot f^{(1)} + 2KD_x g^{(1)}\cdot f^{(1)}, \tag{2.7b}$$

$$\begin{aligned} g_y^{(3)} - f_y^{(3)} - g_{xx}^{(3)} - f_{xx}^{(3)} - 2K(g_x^{(3)} - f_x^{(3)}) = {} & -D_y(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) + D_x^2(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) \\ & + 2KD_x(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}), \ldots \quad \ldots \end{aligned} \tag{2.7c}$$

and

$$g_t^{(1)} - f_t^{(1)} - g_{xxx}^{(1)} + f_{xxx}^{(1)} - 3g_{xy}^{(1)} - 3f_{xy}^{(1)} - 6K(g_{xx}^{(1)} + f_{xx}^{(1)}) - 12K^2(g_x^{(1)} - f_x^{(1)}) = 0, \tag{2.8a}$$

$$\begin{aligned} & g_t^{(2)} - f_t^{(2)} - g_{xxx}^{(2)} + f_{xxx}^{(2)} - 3g_{xy}^{(2)} - 3f_{xy}^{(2)} - 6K(g_{xx}^{(2)} + f_{xx}^{(2)}) - 12K^2(g_x^{(2)} - f_x^{(2)}) \\ & \quad = -D_t g^{(1)}\cdot f^{(1)} + D_x^3 g^{(1)}\cdot f^{(1)} + 3D_xD_y g^{(1)}\cdot f^{(1)} + 6KD_x^2 g^{(1)}\cdot f^{(1)} + 12K^2 D_x g^{(1)}\cdot f^{(1)}, \end{aligned} \tag{2.8b}$$

$$\begin{aligned} & g_t^{(3)} - f_t^{(3)} - g_{xxx}^{(3)} + f_{xxx}^{(3)} - 3g_{xy}^{(3)} - 3f_{xy}^{(3)} - 6K(g_{xx}^{(3)} + f_{xx}^{(3)}) - 12K^2(g_x^{(3)} - f_x^{(3)}) \\ & \quad = -D_t(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) + D_x^3(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) + 3D_xD_y(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) \\ & \quad\quad + 6KD_x^2(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}) + 12K^2 D_x(g^{(1)}\cdot f^{(2)} + g^{(2)}\cdot f^{(1)}), \ldots \quad \ldots \end{aligned} \tag{2.8c}$$

3. Solutions of the KP equation

In this section we are going to derive some exact solutions for the KP equation from the new form BT.