Available online at www.sciencedirect.com
SCIENCE DIRECT®
Chaos, Solitons and Fractals 25 (2005) 481–490
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Interactions among different types of solitary waves in (2 + 1)-dimensional system

Cheng-lin Bai [a,*], Cheng-jie Bai [b], Hong Zhao [a]

[a] Physics Science and Information Engineering School, Liaocheng University, Shandong 252059, China
[b] Propagation School, Shandong Normal University, Shandong 250014, China

Accepted 23 November 2004

Communicated by Prof. M. Wadati

Abstract

Starting from a quite universal formula, which is valid for some quite universal (2 + 1)-dimensional physical models, the interactions among different types of solitary waves like peakons, dromions, and compactons are investigated both analytically and graphically. Some novel features or interesting behaviors are revealed. The results show that the interactions for peakon–antidromion, compacton–antidromion, and antipeakon–compacton may be completely inelastic or not completely elastic.

1. Introduction

In the study of non-linear science, soliton theory plays a very important role and has been applied in almost all the natural sciences especially in all the physics branches such as condensed matter physics, field theory, fluid dynamics, plasma physics and optics, etc. [1]. (1 + 1)-Dimensional solitons and solitary wave solutions have been studied quite well both theoretically and experimentally, for example, dromion (exponentially localized in all directions), compacton and peakon (two types of significant weak solutions). From the symmetry study of the (2 + 1)-dimensional integrable models we know that there exist richer symmetry structures than in lower dimensions [2]. This suggests that the solitary waves structures and the interactions between solitary waves of the (2 + 1)-dimensional non-linear models may have quite rich phenomena that have not yet been revealed. Almost all the previous studies of the interactions among solitary waves especially in higher dimensions are restricted to the same types of localized structures, such as the peakon–peakon and dromion-dromion interactions have been investigated and the results indicate that the former is not completely elastic while the latter is completely elastic, and that for some types of compactons the interactions among them are not completely elastic while for some others the interactions are completely elastic, but the interactions among different types of solitary waves like peakon–antidromion, compacton–antidromion, and antipeakon–compacton are not yet studied too often.

* Corresponding author.
E-mail addresses: lcced_bcl@lctu.edu.cn, lcced_bcl@hotmail.com (C.-l. Bai).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2004.11.021

Motivated by these reasons, we take a (2 + 1)-dimensional breaking soliton system

$$u_t + bu_{xxy} + 4buv_x + 4bu_x v = 0, \tag{1}$$

$$v_x - u_y = 0, \tag{2}$$

where b is an arbitrary constant. Eqs. (1) and (2) describe the interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis and it seems to have been investigated extensively where overlapping solutions have been derived. Since a detailed physical background of the system has been given in Ref. [3], we neglect the corresponding description.

The paper is organized as follows. In Section 2, we apply a variable separation approach to solve the (2 + 1)-dimensional soliton equation and obtain its exact excitation. In Section 3, we analyze interaction properties among different types of coherent soliton structures. A brief discussion and summary is given in the last section.

2. Variable separated solutions for the (2 + 1)-dimensional breaking soliton equation

There was a wealth of approaches for finding special solutions of the non-linear partial differential equation (PDE), such as inverse scattering method, Backlund transformation, Darboux transformation, Hirota method. All these methods are described in Ref. [4–7]. Recently, Lou and coworkers [8,9] has proposed a multi-linear variable separation approach (MLVSA) to search for the exact solutions of the higher-dimensional, specially (2 + 1)-dimensional, non-linear PDEs. Now we apply the MLVSA for the (2 + 1)-dimensional breaking soliton equation.

According to the standard truncated Painleve expansion, we take the following Backlund transformation to u and v in Eqs. (1) and (2)

$$u = \sum_{i=0}^{M} u_i f^{i-M}, \quad v = \sum_{j=0}^{N} v_j f^{j-N}, \tag{3}$$

where u_M and v_N are the arbitrary seed solutions of the breaking soliton system. By using the leading term analysis, we obtain

$$M = 2, \quad N = 2. \tag{4}$$

Substituting Eqs. (3) and (4) directly into Eqs. (1) and (2) and considering the fact that the function u_2 and v_2 are the seed solutions of the original model yield

$$\sum_{i=0}^{4} P_{1i} f^{i-5} = 0, \quad \sum_{i=0}^{2} P_{2i} f^{i-3} = 0, \tag{5}$$

where P_{1i} and P_{2i} are the functions of $\{u_j, v_j, f, j = 0, 1\}$ and their derivatives. Because of the complexity of the expression of P_{1i} and P_{2i}, we neglect their concrete forms. Vanishing the leading and subleading terms of Eq. (5), we can determine the functions $\{u_j, v_j, j = 0, 1\}$. Inserting all the results into Eq. (3) and rewriting its form, we can obtain the Backlund transformation

$$u = \frac{3}{2}(\ln f)_{xx} + u_2, \quad v = \frac{3}{2}(\ln f)_{xy} + v_2. \tag{6}$$

For convenience of discussion, we choose the seed solutions u_2 and v_2 as

$$u_2 = u_{20}(x), \quad v_2 = 0, \tag{7}$$

where $u_{20}(x)$ is an arbitrary function of indicated argument.

Now substituting Eq. (6) together with Eq. (7) into Eq. (1) yields a trilinear equation in f while Eq. (2) is satisfied identically under the above transformation, which reads

$$\begin{aligned}&(4b(u_{20}f_{xy})_x + bf_{xxxxy} + f_{xxt})f^2 + ((2f_{xxx}f_{xy} - f_{xxxx}f_y - 8u_{20}f_xf_{xy} - 4f_{xxxy}f_x + 4(u_{20}f_x)_xf_y)b - 2f_xf_{xt} - f_{xx}f_t)f \\ &\quad + 2((3bf_{xxy} + 4bu_{20}f_y + f_t)f_x^2 + b(f_{xxx}f_y - 3f_{xx}f_{xy})f_x) = 0.\end{aligned} \tag{8}$$

After obtaining the trilinear equation of the original system, we select an appropriate variable separation hypothesis for the function f. For many integrable models, it can be chosen as a modifying Horita's multi-soliton form [10]. In our special case, we take f as a slightly changed as