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On the stratified classical configuration space of lattice QCD

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Abstract

The stratified structure of the configuration space $\mathbf{G}^N = G \times \cdots \times G$ reduced with respect to the action of G by inner automorphisms is investigated for $G = \mathrm{SU}(3)$. This is a finite dimensional model coming from lattice QCD. First, the stratification is characterized algebraically, for arbitrary N. Next, the full algebra of invariants is discussed for the cases $N = 1$ and $N = 2$. Finally, for $N = 1$ and $N = 2$, the stratified structure is investigated in some detail, both in terms of invariants and relations and in more geometric terms. Moreover, the strata are characterized explicitly using local cross sections.

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1. Introduction

If one wants to analyze the non-perturbative structure of gauge theories, one should start with clarifying basic structures like that of the field algebra, the observable algebra

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and the superselection structure of the Hilbert space of physical states. It is clear that the standard Doplicher–Haag–Roberts theory [1,2] for models, which do not contain massless particles, does not apply here. Nonetheless, there are interesting partial results within the framework of general quantum field theory both for quantum electrodynamics (QED) and for non-abelian models, see [3–6].

To be rigorous, one can put the system on a finite lattice, leaving the (extremely complicated task) of constructing the full continuum limit, for the time being, aside. This way, one gets rid of complicated functional analytical problems, but the gauge theoretical problems one is interested in are still present within this setting. For basic notions concerning lattice gauge theories (including fermions) we refer to [7] and references therein. Our approach is Hamiltonian, thus, we put the model on a finite (regular) cubic lattice. In this context, we have formulated (and in the meantime partially solved [8–12]) the following programme:

1. Describe the field algebra $\mathfrak{A}_\Lambda$ in terms of generators and defining relations and endow it with an appropriate functional analytical structure.
2. Describe the observable algebra $\mathfrak{O}_\Lambda$ (algebra of gauge invariant operators, fulfilling the Gauss law) in terms of generators and relations.
3. Analyze the mathematical structure of $\mathfrak{O}_\Lambda$ and endow it with an appropriate functional analytical structure.
4. Classify all irreducible representations of $\mathfrak{O}_\Lambda$.
5. Investigate dynamics in terms of observables.

Finally, of course, one wants to construct the continuum limit. As already mentioned, in full generality, this is an extremely complicated problem of constructive field theory. However, the results obtained until now suggest that there is some hope to control the thermodynamical limit, see [8] for a heuristic discussion. We also mention that for simple toy models, these problems can be solved, see [14].

In [12,13] we have started to investigate the structure of the field and the observable algebra of lattice QCD. In these papers we took the attitude of implementing the constraints on the quantum level. It is well known that there is another possibility: first, one reduces the classical phase space and then one formulates the quantum theory over this reduced phase space. Since the action of the gauge group can have several orbit types, the first step has to be done using singular Marsden–Weinstein reduction [19]. Then the reduced phase space has the structure of a stratified symplectic space. Quantization procedures for such spaces have been worked out recently or are still under investigation [20]. As an important ingredient for both reduction and quantization, in this paper, we study the stratified structure of the reduced classical configuration space. For QCD on a finite lattice, this is given by the orbit space of the action of SU(3) on $\mathrm{SU}(3)^N = \mathrm{SU}(3) \times \cdots \times \mathrm{SU}(3)$ by inner automorphisms.

Our paper is organized as follows: in Section 2 we give a precise formulation of the problem and we discuss the basic tools used in this paper. In Section 3, the stratification of the reduced configuration space is characterized algebraically for arbitrary N. Next, in Section 4 the full algebra of invariants is discussed for the cases $N = 1$ and $N = 2$. Finally, in Sections 5 and 6 the stratified structure is investigated for $N = 1$ and $N = 2$ in some detail, both in terms of invariants and relations and in more geometric terms. Moreover, the strata are characterized explicitly using local cross sections.