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Contact manifolds and generalized complex structures

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Abstract

We give simple characterizations of contact 1-forms in terms of Dirac structures. We also relate normal almost contact structures to the theory of Dirac structures.
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1. Introduction

Dirac structures on manifolds provide a unifying framework for the study of many geometric structures such as Poisson structures and closed 2-forms. They have applications to modeling of mechanical and electrical systems (see, for instance, [2]). Dirac structures were introduced by Courant and Weinstein (see [3,4]). Later, the theory of Dirac structures and Courant algebroids was developed in [11].

In [7], Hitchin defined the notion of a generalized complex structure on an even-dimensional manifold M, extending the setting of Dirac structures to the complex vector

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bundle $(TM \oplus T^*M) \otimes \mathbb{C}$. This allows to include other geometric structures such as Calabi-Yau structures in the theory of Dirac structures. Furthermore, one gets a new way to look at Kähler structures (see [6]). However, the odd-dimensional analogue of the concept of a generalized complex structure was still missing. The aim of this Note is to fill this gap.

The first part of this paper concerns characterizations of contact 1-forms using the notion of an $\mathcal{E}^1(M)$-Dirac structure as introduced in [12]. In the second part, we define and study the odd-dimensional analogue of a generalized complex structure, which includes the class of almost contact structures. There are many distinguished subclasses of almost contact structures: contact metric, Sasakian, K-contact structures, etc. We hope that the theory of Dirac structures will lead to new insights on these structures.

2. $\mathcal{E}^1(M)$-Dirac structures

2.1. Definition and examples

In this section, we recall the description of several geometric structures (e.g. contact structures) in terms of Dirac structures.

First of all, observe that there is a natural bilinear form $\langle\cdot,\cdot\rangle$ on the vector bundle $\mathcal{E}^1(M) = (TM \times \mathbb{R}) \oplus (T^*M \times \mathbb{R})$ defined by:

$$\langle (X_1, f_1) + (\alpha_1, g_1), (X_2, f_2) + (\alpha_2, g_2)\rangle = \frac{1}{2}(i_{X_2}\alpha_1 + i_{X_1}\alpha_2 + f_1 g_2 + f_2 g_1)$$

for any $(X_j, f_j) + (\alpha_j, g_j) \in \Gamma(\mathcal{E}^1(M))$, with $j = 1, 2$. Moreover, for any integer $k \geq 1$, one can define

$$\tilde{d} : \Omega^k(M) \times \Omega^{k-1}(M) \to \Omega^{k+1}(M) \times \Omega^k(M),$$

by the formula

$$\tilde{d}(\alpha, \beta) = (\mathrm{d}\alpha,\ \alpha - \mathrm{d}\beta)$$

for any $\alpha \in \Omega^k(M)$, $\beta \in \Omega^{k-1}(M)$, where d is the exterior differentiation operator. When $k = 0$, we define $\tilde{d}f = (\mathrm{d}f, f)$. Clearly, $\tilde{d}^2 = 0$. We also have the contraction map given by:

$$i_{(X,f)}(\alpha, \beta) = (i_X\alpha + f\beta,\ -i_X\beta)$$

for any $X \in \mathfrak{X}(M)$, $f \in C^\infty(M)$, $\alpha \in \Omega^k(M)$, $\beta \in \Omega^{k-1}(M)$. From these two operations, we get

$$\tilde{\mathcal{L}}_{(X,f)} = i_{(X,f)} \circ \tilde{d} + \tilde{d} \circ i_{(X,f)}.$$

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