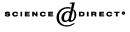


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Journal of Geometry and Physics 53 (2005) 31-48



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## On the geometry and homology of certain simple stratified varieties

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Received 31 March 2003; received in revised form 28 April 2004; accepted 28 April 2004 Available online 19 June 2004

## Abstract

We study certain mild degenerations of algebraic varieties which appear in the analysis of a large class of supersymmetric theories, including superstring theory. We analyze Witten's  $\sigma$ -model [Nucl. Phys. B 403 (1993) 159] and find that the non-transversality of the superpotential induces additional singularities and a stratification of the ground state variety. This stratified variety admits certain homology groups such that  $\bigoplus_q H^{2q}$  satisfies the "Kähler package" of requirements [Ann. Math. Studies 102 (1982) 303]. Also, this  $\bigoplus_q H^{2q}$  extends the "flopped" pair of small resolutions [Nucl. Phys. B 416 (1994) 414; Nucl. Phys. B 330 (1990) 49; Commun. Math. Phys. 119 (1988) 431] to an "(exo)flopped" triple, and is compatible with both mirror symmetry [S.-T. Yau (Ed.), Mirror Manifolds, International Press, Hong Kong, 1990; B. Greene, S.-T. Yau (Eds.), Mirror Manifolds II, International Press, Hong Kong, 1996] and string theory [Mod. Phys. Lett. A 12 (1997) 521; Nucl. Phys. B 451 (1995) 96] results. Finally, we revisit the conifold transition [Nucl. Phys. B 330 (1990) 49] as it applies in our formalism.

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MSC: 14F43; 14J32

JGP SC: Differential geometry; Strings

Keywords: Stratified varieties; Superstring theory;  $\sigma$ -Model

## 1. Introduction, results and summary

In string theory, rather than being an assumed arena, the spacetime is identified with the dynamically determined 'ground state variety' of a (supersymmetric)  $\sigma$ -model [11,20,23].

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In the simplest physically interesting and non-trivial case [3,23], the spacetime is of the form  $M^{3,1} \times K$ , where *K* is a compact Calabi-Yau three-fold modeled from the (bosonic subset of the) 'field space' of the  $\sigma$ -model<sup>1</sup>,  $\mathcal{F} = \{p, s_0, \ldots, s_4\} \simeq \mathbb{C}^6$ , which admits a  $\mathbb{C}^*$  action:

$$\hat{\lambda}: \{p, s_0, \dots, s_4\} \mapsto \{\lambda^{-5} p, \lambda s_0, \dots, \lambda s_5\}, \quad \lambda \in \mathbb{C}^*.$$
(1)

The 'ground state variety' is defined to be [2,23]

$$\mathcal{V} \stackrel{\text{def}}{=} [(\partial W)^{-1}(0) - 0] / \hat{\lambda}, \tag{2}$$

with the  $\hat{\lambda}$ -invariant holomorphic 'superpotential'

$$W \stackrel{\text{def}}{=} p \cdot G(s). \tag{3}$$

Alternatively, we denote by  $\hat{\lambda}$  the  $|\lambda| = 1$  restriction of the map (1), and define the 'potential'

$$U_r \stackrel{\text{def}}{=} \|\partial W\|^2 + D_r^2,\tag{4}$$

where

$$D_r \stackrel{\text{def}}{=} ||s||^2 - 5|p|^2 - r, \quad r \in \mathbb{R}.$$
 (5)

Then

$$\mathcal{V} \simeq [U_r^{-1}(0) - 0]/\hat{\lambda}.$$
 (6)

Due to the positive definiteness of  $U_r$ ,

$$U_r^{-1}(0) = (\partial W)^{-1}(0) \cap D_r^{-1}(0).$$
<sup>(7)</sup>

Furthermore, the  $\hat{\lambda}$ -invariance of W = pG implies that G(s) is a degree-5 homogeneous complex polynomial

$$G(\lambda s_0, \dots, \lambda s_4) = \lambda^5 G(s_0, \dots, s_4), \tag{8}$$

whereupon the zero locus of  $\partial W$  is the intersection of the cones

$$(\partial W)^{-1}(0) = G^{-1}(0) \cap (p \cdot \partial_s G)^{-1}(0).$$
(9)

The above definition may then be rephrased as follows.

**Definition 1.** Given the polynomials G(s) and  $D_r$  as defined in Eqs. (8) and (5), respectively, the 'ground state variety' is

$$\mathcal{V} = \{ G^{-1}(0) \cap (p \cdot \partial_s G)^{-1}(0) - 0 \} / \hat{\lambda} = \{ G^{-1}(0) \cap (p \cdot \partial_s G)^{-1}(0) \cap D_r^{-1}(0) - 0 \} / \hat{\lambda},$$
(10)

where the  $S^1$ -action,  $\hat{\lambda}$ , in the latter (symplectic) quotient is the  $|\lambda| = 1$  restriction of the  $\mathbb{C}^*$ -action (1) in the former (holomorphic) quotient.

 $\mathcal{V}^+$  ( $\mathcal{V}^-$ ) shall denote the restriction of  $\mathcal{V}$  to positive (negative) values of r in Eq. (5).

<sup>&</sup>lt;sup>1</sup> To avoid obscuringly complicated notation, we focus on a simple example and discuss generalizations later.

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