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Grazing bifurcations and chatter in a pressure relief valve model

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ABSTRACT

This paper considers a simple mechanical model of a pressure relief valve. For a wide region of parameter values, the valve undergoes self-oscillations that involve impact with the valve seat. These oscillations are born in a Hopf bifurcation that can be either super- or sub-critical. In either case, the onset of more complex oscillations is caused by the occurrence of grazing bifurcations, where the limit cycle first becomes tangent to the discontinuity surface that represents valve contact. The complex dynamics that ensues from such points as the flow speed is decreased has previously been reported via brute-force bifurcation diagrams. Here, the nature of the transitions is further elucidated via the numerical continuation of impacting orbits. In addition, two-parameter continuation results for Hopf and grazing bifurcations as well as the continuation of period-doubling bifurcations of impacting orbits are presented. For yet lower flow speeds, new results reveal chattering motion, that is where there are many impacts in a finite time interval. The geometry of the chattering region is analysed via the computation of several pre-images of the grazing set. It is shown how these pre-images organise the dynamics, in particular by separating initial conditions that lead to complete chatter (an accumulation of impacts) from those which do not.

1. Introduction

The recent extension of classic nonlinear bifurcation theory to piecewise-smooth dynamical systems (see for example [1–5]) has allowed the qualitative analysis of a vast range of systems that arise in engineering. Such systems are often nonsmooth by their very nature due, for example, to impact, backlash, free-play, switching, dry friction or sticking. In addition to all the nonlinear phenomena that smooth dynamical systems can undergo, such as local and global bifurcation of invariant sets, there are a class of phenomena that are unique to piecewise-smooth systems. These have been termed *discontinuity-induced bifurcations* [2,3,6]. They occur when topological equivalence of the phase portrait is lost under parameter variation, in which any discontinuity sets must also be taken into account in the equivalence. Canonical examples of such discontinuity-induced bifurcations occur when an Ω-limit set (for example, a limit cycle) becomes tangent (or *grazes*) with a discontinuity set.

This paper shall specifically concern an example of the so-called impacting class of hybrid systems. Here the phase space is locally a half-space, on the boundary of which, a *reset map* or an *impact law* takes trajectories with positive impacting velocity to those with an outgoing velocity, in instantaneous time. There is a rich literature starting from the 1970s on the simplest forms of such hybrid systems, the so-called impact oscillators; see for example [7–14]. Here, complex dynamics has been found to result from the occurrence of a grazing bifurcation; see [2, Chapter 6] for the relevant theory, mostly due to Nordmark and co-workers, e.g. [15].

A distinction between the dynamics described in this paper and that of most other examples of impact oscillators is that the system in question here is autonomous. That is, the underlying oscillation that undergoes the grazing bifurcation is intrinsic to the system and not due to explicit external driving. Indeed, as we shall show, the system displays many of the possible dynamical features of low degree of freedom impacting systems; in particular, different types of grazing bifurcation and the complex dynamics associated with *chattering* and *sticking* behaviour. Here, chattering, sometimes also referred to as zenoness denotes an accumulation of impact events in finite time, and sticking is where the dynamics becomes constrained to the impact surface (like a tennis ball coming to rest on the racket). Yet, for our example, the dynamical equations contain only three dependent variables. As such, we believe this model could serve as a good tutorial example for studying the complex dynamics due to the impact in an autonomous system.

The model also represents an important physical phenomenon, namely the behaviour of hydraulic pressure relief valves, which are well known to be susceptible to violent oscillation of the valve body

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at low pressures. These oscillations often involve impact which can cause extensive damage to both the valve body and its seat, can contribute to fatigue failure of pressure vessels and piping components and typically produces considerable noise. Indeed, the second author spent a whole year in an office in which the heating valve, due to a plumbing error, continually underwent flow-induced vibrations of such intensity that their volume caused significant impediment even to normal conversation!

Thus, the practical motivation beyond providing a good tutorial example is to explore the influence of the main system parameters on the valve dynamics and to provide stability maps in terms of universal dimensionless parameters. Such maps would ease the design of real-life engineering systems or contribute to unfolding the root of failures a posteriori. Moreover, due to the hysteresis of the primary Hopf bifurcation, a large-amplitude impacting limit cycle is present in the linearly stable parameter range, that cannot be detected with conventional (smooth) techniques. We believe that by computing the global, nonlinear stability boundary we provide a useful tool for pressure relief valve design.

The analysis of pressure valve oscillations has a history that goes back at least to the late 1960s in the work of Kasai [16]. Further work in the 70s, 80s and 90s includes those in Refs. [17–23]. Perhaps the most comprehensive nonlinear analysis is that of Hayashi et al. [24] who finds evidence for chaotic oscillation in a direct-operated relief valve together connected by a pipe to an upstream chamber. By modelling the pipe flow using an unsteady Bernoulli equation, allowing the capacitive characteristic of the chamber to represent a single degree of freedom, with another differential equation for pressure, they arrive at a system of four first-order ODEs for the motion of the valve body. Linear stability analysis was augmented with direct numerical simulation, which revealed a period-doubling route chaos. Another relevant study is that Eyres et al. [25], where the nonlinear dynamics of a hydraulic damper with a "blow-off" pressure valve was studied using a combination of direct simulation, numerical and analytical bifurcation techniques unique to piecewise-smooth systems. Maccari in [26] includes the continuum dynamics of the spring and models the valve seat by a massless spring with nonlinear (smooth) characteristics. Hysteresis and jumps are found in the force–response and frequency–response curves together with saddle–node bifurcations of cycles that may result in unsatisfactory relief valve performance.

The rest of this paper is organised as follows. Section 2 introduces the dimensionless equations of the model system to be studied, along with a precis of the results previously obtained on it by the present authors together with G. Licskó [27]. Section 3 is then devoted to a description of the dedicated numerical methods we use in order to numerically analyse the system, taking care to reflect the nonsmooth nature of the problem. Section 4 presents numerically computed bifurcation diagrams of both smooth and discontinuity-induced bifurcations in both one and two parameters. The physical parameters varied represent the flow rate and the setting of the valve precompression (opening pressure). Section 5 focuses specifically on a geometric interpretation of the chattering-type behaviour that occurs for low flow rates. Finally, Section 6 draws conclusions and suggests avenues for future work.

2. Mathematical model

The model we investigate was introduced in [27] in order to explain experimentally observed oscillations in a simple hydraulic system containing a pressure relief valve. Interested readers are referred to that paper for more on the physical interpretation of the model and the relation of the dimensionless variables and parameters presented here to physical, dimensional quantities.

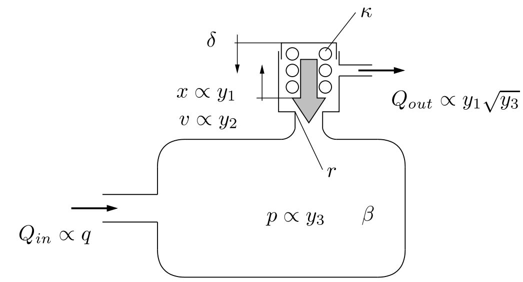

Fig. 1. Sketch of the physical system analysed. Here $y_{1...3}$ stand for the dimensionless displacement, velocity and pressure, q is the dimensionless flow rate entering the system, δ and κ are the (dimensionless) spring precompression and damping coefficients, r is the restitution coefficient between the seat and the valve body, β is a measure of the compressibility parameter of the fluid and the elastic hoses. See [27] for more details.

2.1. Governing equations

Consider the hydrostatic system depicted in Fig. 1. Although the model itself is highly simplified, it captures features that are common to many hydraulic power transmission systems. In particular, the flow rate is assumed to be constant, independent of the actual load applied to the system. Such a constant flow could be maintained by a positive displacement pump, such as a gear pump. Second, the system topology is highly simplified to that of a single reservoir; nevertheless the properties of the fluid in this reservoir can be thought of as representing the compressibility of the hydraulic fluid in a more complex transmission system and the elasticity of the transmission lines. Most crucially, the system contains a pressure relief valve for protection of the system to excess pressure. Finally, for simplicity we suppose that any moving mechanical parts such as hydraulic actuators are at rest, so that the flow rate of the continuously operating pump is exhausted only through the relief valve.

The valve itself is modelled as a rigid poppet that is held closed by a spring. Thus, while the valve is closed, the outflow Q_{out} is zero, whereas the inflow Q_{in} is a positive constant. During this state pressure must build up in the chamber, which will cause the valve to open. Opening the valve causes outflow (typically into a collection tank), which reduces pressure and allows the valve to close again. This, in a nutshell, is the origin of oscillation in the system.

Ignoring any wave or eddy effects in the fluid, the dynamics of the system can be described by three dynamical variables, the position y_1 and velocity y_2 of the poppet, and the pressure in the chamber y_3. The equations of motion for these three quantities take into account the Newtonian mechanics of the valve body and the pressure dynamics of the reservoir. Specifically, the non-dimensional equations can be written in the form

$$y_1' = y_2 \tag{2.1}$$

$$y_2' = -\kappa y_2 - (y_1 + \delta) + y_3 \tag{2.2}$$

$$y_3' = \beta\left(q - y_1\sqrt{y_3}\right), \tag{2.3}$$

provided the valve is open ($y_1 > 0$), coupled to the impact law

$$R : (0, y_2^-, y_3^-) \mapsto (0, y_2^+, y_3^+), \quad \text{where } y_2^+ = -ry_2^-,\ y_3^+ = y_3^-,$$

which applies whenever $y_1 = 0$ for $y_2^- \leq 0$. Here $0 < r < 1$ is a Newtonian coefficient of restitution that approximates the loss of energy in whenever an impact occurs between the valve body and its seat. It is also assumed that $y_3 > 0$, i.e. the reservoir pressure is above the ambient pressure thus the flow direction is always outwards from the reservoir.

The details of the derivation, including the underlying physical assumptions can be found in [27], here we describe only the