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Multiple structures of two-dimensional nonlinear Rossby wave

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Abstract

In this paper, the elliptic equation is taken as a transformation and applied to solve the Zakharov–Kuznetsov equation, which has been derived by Gottwald as a two-dimensional model for nonlinear Rossby waves. It is shown that more kinds of solutions are derived, such as periodic solutions of rational form, periodic solutions and so on.

1. Introduction

Gottwald derived the nonlinear dispersive Zakharov–Kuznetsov equation from the quasigeostrophic barotropic vorticity equation [1], where weakly nonlinear and long wave multiple scale analysis were applied to two dimensions and derived the Zakharov–Kuznetsov (ZK for short) equation for nonlinear Rossby waves. In contrast to the Kadometsev–Petviashvilli (KP for short) equation, the ZK equation is first derived in a geophysical fluid dynamics context. At the same time, the ZK equation supports stable lump solitary waves [2], which makes the ZK equation a very attractive model equation for the study of vortices in geophysical flows. Actually, there are more multiple structures in the ZK equations, we will show next. All these studies may help to describe two-dimensional coherent structures such as atmospheric blocking events, long lived eddies in the ocean or coherent structures in the Jovian atmosphere such as the Great Red Spot.

We have taken elliptic equation as an intermediate transformation to solve nonlinear wave equations [3–5], and obtained many periodic solutions and solitary wave solutions. However, there are still more research needed to do in order to find more solutions of different forms. In the reference [6], we derived periodic solutions of rational forms, which are due to external forcing. In this paper, the elliptic equation is taken as a transformation and applied to solve the ZK equation to multiple structures of two-dimensional nonlinear Rossby waves.

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2. Multiple structures of two-dimensional nonlinear Rossby waves

ZK equation [1] reads

$$u_t + \delta u_x + \alpha u u_x + \beta u_{xxx} + \gamma u_{xyy} = 0 \tag{1}$$

which was derived by Gottwald from the quasigeostrophic vorticity equation as a two-dimensional model for nonlinear Rossby waves.

We seek its travelling wave solutions in the following frame

$$u = u(\xi), \quad \xi = kx + ly - \omega t \tag{2}$$

here ω is angular frequency, k and l are wave number in x and y direction, respectively.

Substituting Eq. (2) into Eq. (1) and integrating once yield

$$-(\omega - \delta k)u + \frac{\alpha k}{2}u^2 + (\beta k^3 + \gamma k l^2)u'' = C \tag{3}$$

where C is an integration constant. And then we suppose Eq. (3) has the following solution

$$u = u(z) = \sum_{j=0}^{j=n} b_j z^j, \quad z = z(\xi), \quad b_n \neq 0 \tag{4}$$

where z satisfies the elliptic equation [7]

$$z'^2 = \sum_{i=0}^{i=4} a_i z^i, \quad a_4 \neq 0 \tag{5}$$

where $z' = \frac{dz}{d\xi}$, then

$$z'' = \frac{a_1}{2} + a_2 z + \frac{3a_3}{2}z^2 + 2a_4 z^3 \tag{6}$$

Obviously, two special cases of (5) are

$$\frac{dz}{d\xi} = b + z^2 \tag{7}$$

and

$$\frac{dz}{d\xi} = R(1 + \mu z^2) \tag{8}$$

which were introduced by Fan [8] and Yan et al. [9], respectively.

There n in the Eq. (4) can be determined by the partial balance between the highest order derivative terms and the highest degree nonlinear term in the Eq. (3). Here we know that the degree of u is

$$O(u) = O(z^n) = n \tag{9}$$

and from (5) and (6), one has

$$O(z'^2) = O(z^4) = 4, \quad O(z'') = O(z^3) = 3 \tag{10}$$

and actually one can have

$$O(z^{(d)}) = d + 1 \tag{11}$$

So one has

$$O(u) = n, \quad O(u') = n + 1, \quad O(u'') = n + 2, \quad O(u^{(d)}) = n + d \tag{12}$$

For ZK equation (1), we have $n = 2$, so the ansatz solution of (4) can be rewritten as

$$u = b_0 + b_1 z + b_2 z^2, \quad b_2 \neq 0 \tag{13}$$

then

$$u^2 = b_0^2 + 2b_0 b_1 z + (2b_0 b_2 + b_1^2)z^2 + 2b_1 b_2 z^3 + b_2^2 z^4 \tag{14}$$