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New exact solutions to the KdV–Burgers–Kuramoto equation

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Abstract

New exact traveling wave solutions to the KdV–Burgers–Kuramoto equation (thereafter KBK equation) are obtained by using trigonometric function expansion method. They are compared with the solutions deduced from other methods.

1. Introduction

Many phenomena are simultaneously involved in nonlinearity, dissipation, dispersion and instability. As Kuramoto [1] suggested that KBK equation [2]:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\alpha\frac{\partial^2 u}{\partial x^2}+\beta\frac{\partial^3 u}{\partial x^3}+\gamma\frac{\partial^4 u}{\partial x^4}=0 \tag{1}$$

is an appropriate model to describe these phenomena, where α, β and γ are constants. Eq. (1) is also known as the Kuramoto–Sivashinsky equation [3] or Benney equation [2].

In order to well understand various nonlinear phenomena, many methods for obtaining analytical solutions of nonlinear evolution equations have been proposed, among them are Hirota's dependent variable transformation [4], the inverse scattering method [4], homogeneous balance method [5,11], trial-function method [2], trigonometric function method [6], tanh-function method [7], truncated expansion method [8] and so on. The solutions of KBK equation possesses their actual physical application, this is the reason why so many methods, such as Weiss–Tabor–Carnevale transformation method [3], trial-function method [2], tanh-function method [7], homogeneous balance method [9] and so on, have been applied to obtained the solution to KBK equation. But no method is both convenient and able to be used to get as many solutions as possible. Weiss–Tabor–Carnevale transformation method, tanh-function method and homogeneous balance method are complicated in deriving the solutions to KBK, no explicit solution and parameter constraint were given in Ref. [7], and only one special case was considered in Ref. [9], so only a special solution was given there. Trial-function method is a simple one, some solutions can be easily obtained, too. But some solution cannot be got with this method, either. In this paper, trigonometric function expansion method is used to obtain solutions to KBK equation and comparison with solutions got in Refs. [2,3,7,9] are given.

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2. Solutions to KBK equation

In order to solve Eq. (1), the following transformation:

$$\xi = k(x - ct) \tag{2}$$

is needed, where k is called wave number and c is wave speed.

Substitution (2) into Eq. (1) and integrating it once lead to

$$-cu + \frac{1}{2}u^2 + \alpha k \frac{du}{d\xi} + \beta k^2 \frac{d^2u}{d\xi^2} + \gamma k^3 \frac{d^3u}{d\xi^3} = A \tag{3}$$

where A is a constant of integration.

Based on the trigonometric function expansion method [6], Eq. (3) may have the following ansatz solution:

$$u(\xi) = \sum_{j=1}^{n} (a_j \sin\omega + b_j \cos\omega)\sin^{j-1}\omega + a_0$$

where ω satisfies relation

$$\frac{d\omega}{d\xi} = \sin\omega \tag{4}$$

And n can be determined by partially balancing the highest degree nonlinear term and the derivative terms of high order in Eq. (3), here it is determined as $n = 3$. So the ansatz solution takes the following form:

$$u = a_0 + a_1 \sin\omega + a_2 \sin^2\omega + a_3 \sin^3\omega + \cos\omega(b_1 + b_2 \sin\omega + b_3 \sin^2\omega) \tag{5}$$

Here stress is needed to lay on the coefficients b_j $(j = 1, 2, 3)$, in order to get nontrivial solutions to Eq. (3), b_3 cannot be set as zero. Similarly, if we set all b_j $(j = 1, 2, 3)$ as zeros, no nontrivial solutions can be obtained.

From the ansatz solution (5), the following relations can be easily got:

$$\frac{du}{d\xi} = b_2 \sin\omega + (2b_3 - b_1)\sin^2\omega - 2b_2 \sin^3\omega - 3b_3 \sin^4\omega + \cos\omega(a_1 \sin\omega + 2a_2 \sin^2\omega + 3a_3 \sin^3\omega) \tag{6}$$

$$\begin{aligned}\frac{d^2u}{d\xi^2} = {} & a_1 \sin\omega + 4a_2 \sin^2\omega + (9a_3 - 2a_1)\sin^3\omega - 6a_2 \sin^4\omega - 12a_3 \sin^5\omega + \cos\omega[b_2 \sin\omega + 2(2b_3 - b_1) \\ & \times \sin^2\omega - 6b_2 \sin^3\omega - 12b_3 \sin^4\omega]\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{d^3u}{d\xi^3} = {} & b_2 \sin\omega + 4(2b_3 - b_1)\sin^2\omega - 20b_2 \sin^3\omega - 6(10b_3 - b_1)\sin^4\omega + 24b_2 \sin^5\omega + 60b_3 \sin^6\omega \\ & + \cos\omega[a_1 \sin\omega + 8a_2 \sin^2\omega + (27a_3 - 6a_1)\sin^3\omega - 24a_2 \sin^4\omega - 60a_3 \sin^5\omega]\end{aligned} \tag{8}$$

$$\begin{aligned}u^2 = {} & (a_0^2 + b_1^2) + (2a_0a_1 + 2b_1b_2)\sin\omega + (2a_0a_2 + a_1^2 + b_2^2 + 2b_1b_3 - b_1^2)\sin^2\omega + 2(a_0a_3 + a_1a_2 \\ & + b_2b_3 - b_1b_2)\sin^3\omega + (2a_1a_3 + a_2^2 + b_3^2 - b_2^2 - 2b_1b_3)\sin^4\omega + 2(a_2a_3 - b_2b_3)\sin^5\omega + (a_3^2 - b_3^2)\sin^6\omega \\ & + 2\cos\omega[a_0b_1 + (a_1b_1 + a_0b_2)\sin\omega + (a_2b_1 + a_1b_2 + a_0b_3)\sin^2\omega + (a_3b_1 + a_2b_2 + a_1b_3)\sin^3\omega \\ & + (a_3b_2 + a_2b_3)\sin^4\omega + a_3b_3 \sin^5\omega]\end{aligned} \tag{9}$$

So substituting (5)–(9) into (3) results in a algebraic equation about expansion coefficients a_j and b_j. Setting the coefficients of various $\sin^j\omega$ $(j = 0, 1, \ldots, 6)$ and $\cos\omega\sin^j\omega$ $(j = 0, 1, \ldots, 5)$ as zeros, one can obtain a set of algebraic equations about the expansion coefficients a_j and b_j, i.e.

$$-ca_0 + \frac{1}{2}(a_0^2 + b_1^2) = A$$

$$-ca_1 + (a_0a_1 + b_1b_2) + \alpha k b_2 + \beta k^2 a_1 + \gamma k^3 b_2 = 0$$

$$-ca_2 + \frac{1}{2}(2a_0a_2 + a_1^2 + b_2^2 + 2b_1b_3 - b_1^2) + \alpha k(2b_3 - b_1) + 4\beta k^2 a_2 + 4\gamma k^3(2b_3 - b_1) = 0$$