ON THE EXPONENTIAL DECAY OF MAGNETIC STARK RESONANCES

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We study the time decay of magnetic Stark resonant states. As a main result we prove that for sufficiently large time these states decay exponentially with the rate given by the imaginary parts of eigenvalues of certain non-selfadjoint operator. The proof is based on the method of complex translations.

Keywords: Stark Hamiltonians, resonances, complex translations.

1. Introduction

The purpose of this paper is to study the time decay of resonances in two dimensions in the presence of crossed magnetic and electric fields and a potential type perturbation. Such a system was mathematically studied in [15] using analytic dilations and most recently also in [5].

We assume that the magnetic field points in the direction perpendicular to the electron plane with a constant intensity B and that the electric field of constant intensity F points in the x direction. The perturbation $V(x, y)$ is supposed to satisfy certain localization conditions. The corresponding quantum Hamiltonian reads as follows:

$$H(F) = H(0) - Fx = H_L + V - Fx,$$

where H_L is the Landau Hamiltonian of an electron in a homogeneous magnetic field B.

We begin with the notion of a resonance in terms of a complex eigenvalue of certain non-selfadjoint operator. In Section 3 we show the connection between the

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imaginary parts of these eigenvalues and the time decay of the matrix elements of $e^{-itH(F)}$.

The central mathematical tool, that we use, is the method of complex translations for Stark Hamiltonians, which was introduced in [2] as a modification of the theory of complex scaling [1–3]. Following [2] we consider the transformation $U(\theta)$, which acts as a translation in x direction, $(U(\theta)\psi)(x) = \psi(x+\theta)$. For a nonreal θ the translated operator $H(F,\theta) = U(\theta)H(F)U^{-1}(\theta)$ is non-selfadjoint and therefore can have some complex eigenvalues. The main result of Section 3, Theorem 3.1, tells us that the matrix elements of $e^{-itH(F)}$ decay exponentially at the rate given by the imaginary parts of the eigenvalues of $H(F,\theta)$. Thus our result can be regarded as a generalization of the result obtained in [10], where the exponential decay was proved for the Stark Hamiltonians without magnetic field.

In the proof of Theorem 3.1 we make use of the explicit formula for the integral kernel of the operator e^{-itH_1}, which is defined by Eq. (2.1) below. We have calculated the kernel using the functional integration method and verified the correctness of the result, see Appendix.

2. The model

We work in the system of units, where $m = 1/2$, $e = 1$, $\hbar = 1$. The crossed fields Hamiltonian is then given by

$$H_1(F) = H_L - Fx = (-i\partial_x + By)^2 - \partial_y^2 - Fx, \qquad \text{on} \quad L^2(\mathbb{R}^2). \tag{2.1}$$

Here we use the Landau gauge where $\mathbf{A}(x,y) = (-By, 0)$. A straightforward application of [13, Theorem X.37] shows that $H_1(F)$ is essentially self-adjoint on $C_0^\infty(\mathbb{R}^2)$, see also [13, Problem X.38]. Moreover, one can easily check that

$$\sigma(H_1(F)) = \sigma_{\mathrm{ac}}(H_1(F)) = \mathbb{R}. \tag{2.2}$$

As mentioned in Introduction we employ the translational analytic method developed in [2]. We introduce the translated operator $H_1(F,\theta)$ as follows:

$$H_1(F,\theta) = U(\theta)H_1(F)U^{-1}(\theta), \tag{2.3}$$

where

$$(U(\theta)f)(x,y) := \left(e^{ip_x\theta} f\right)(x,y) = f(x+\theta, y). \tag{2.4}$$

Elementary calculation shows that

$$H_1(F,\theta) = H_1(F) - F\theta. \tag{2.5}$$

The operator $H_1(F,\theta)$ is clearly analytic in θ.

Now we formulate the conditions to be imposed on V:

(a) For some $\beta_0 > 0$ independent of y the function $V(x+z,y)$ is analytic and uniformly bounded in the strip $|\Im z| \le \beta_0$. In addition

$$\lim_{|(x,y)|\to\infty} |V(x+z,y)| = 0.$$

(b) The operator $H(F) = H_1(F) + V$ has purely continuous spectrum.