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High frequency vibroacoustics: A radiative transfer equation and radiosity based approach

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HIGHLIGHTS

- Radiant energy exchange in structural dynamics and acoustics.
- Application to the radiosity method in acoustics.
- Example of specular reflection.
- Application to multiple diffraction.

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ABSTRACT

This paper is a review of the theoretical framework for the application of radiative transfer equations to structural dynamics and acoustics. It is shown that under the assumption of geometrical acoustics and when the phase of rays is neglected, a representation of a sound field in terms of incoherent fictitious sources provides a sufficiently large framework to embody diffuse as well as specular reflection, steady-state or transient phenomena and even diffraction. Two special cases are mentioned. The so-called radiosity method in acoustics corresponds to a purely diffusing boundary and statistical energy analysis when the sound field is diffuse.

1. Introduction

The interest of the scientific community in high frequency methods applied to vibro-acoustics is relatively recent compared to other fields of physics such as electrodynamics or acoustics. It seems that ray methods have not been as popular in structural dynamics where most engineers prefer to use softwares based on the governing equations solved by the finite element method.

However, this 'brute force' approach of vibro-acoustics rapidly encounters a practical limitation. When the frequency increases, the number of natural modes increases. Even for a numerical model of order of tens of millions of degrees of freedom, a reasonable task on current computers, the maximum analysis frequency is still too low and does not permit the entire audio frequency band to be covered. This limitation is particularly unfavorable in the automotive industry or for larger structures such as aircraft or launchers which may contain millions of natural modes within the audio frequency band [1]. Furthermore, it might be emphasized that the sensitivity of these computations is poor which raises the question of how useful it is to know the 'exact' sequence of natural modes for systems having a large number of modes.

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For these reasons, statistical energy analysis [2] (SEA) has been developed for fifty years. Introduced early in the sixties by R.H. Lyon [3] and many other authors [4–6], this theory is based on statistical concepts and applies when sources are random, uncorrelated and when the number of modes is large enough to be considered as a statistical population. Then under certain circumstances [7], sound/vibration reaches a state of diffuse field and equipartition of modal energy (an interesting method to dispense with equipartition is proposed in Refs. [8,9]). Then, a simple law emerges: the energy exchange between two coupled subsystems is proportional to the difference in their modal energies. This has served as a foundation to interpret the modal energy as a 'vibrational temperature' and the coupling power proportionality as a thermal law. SEA follows on from room acoustics and Sabine's formula and as such could be considered as a theory of statistical vibroacoustics [10] in the same way as statistical mechanics, statistical electromagnetism and so on.

Among numerous attempts to relax the more restrictive assumptions of SEA, several methods aim to predict the repartition of vibrational energy within subsystems where the field is not diffuse. But the behavior of local energy quantities is more subtle than appears at first sight [11–13] and it is generally not possible to obtain a closed set of equations involving energy variables without introducing strong assumptions.

To circumvent this difficulty, it has been proposed by several authors [14–25] to start from an integral representation of the vibrational field and to calculate a quadratic quantity by neglecting to various degrees the coherence between the boundary sources. When all sources are incoherent, it results in a boundary integral equation for energy that may be solved by a collocation or the Galerkin method. Although they have been developed in various contexts, all these methods suggest more or less the idea that vibrational energy propagates in structures like radiant thermal energy. This thermal analogy is clearly not equivalent to the thermal conductivity proposed in other respects (see Refs. [26,27,23] for a discussion on this point). The radiative transfer analogy is more apparent in room acoustics with the so-called radiosity method [28–32]. Furthermore, when the vibrational field tends to be diffuse, it is found that the coupling power proportionality of statistical energy analysis is obtained as a limiting case [33]. In this regard, let us also mention the recently proposed method called dynamical energy analysis which investigates the gap between full ray-tracing and statistical energy analysis [34–36].

The purpose of this paper is to review the theoretical framework of an extension of the radiosity method to specular reflection and diffraction which may serve as a basis for a generalization of statistical energy analysis. The outline of the paper is as follows. In Section 2 we introduce some generalities about representation of a vibration field in terms of radiant energy supplied by actual and fictitious sources. In Section 3, the balance equations are derived governing the emissive power of fictitious sources. The special case of diffuse reflection and the radiosity equation is introduced in Section 4. Two problems which enlarge the scope of classical radiosity are then discussed, specular reflection in Section 5 and diffraction in Section 6. The paper finishes with some concluding remarks.

2. Energy representation of a wave field

The laws of radiative energy transfer are based on two principles; the linear superposition principle and Huygens' principle.

If the vibrational field is sufficiently 'disordered' then we may neglect all correlations between rays and energy variables turn out to be well-suited to describe the field state. Two rays emanating from a single source with a slightly different direction take different paths and are subjected to different events such as reflection, diffraction, diffusion *etc.* before eventually reaches the same receiver point. They rapidly 'forget' their phase so that after traveling a long distance, their energies may be added signifying that interference effects can be neglected.

Huygens' principle states that each point of reflection, refraction, diffraction *etc.* behaves like a secondary source from which emanate spherical waves. A more modern form is the so-called Helmholtz–Kirchhoff formula or any representation formula by Green's function. The principle of all these formulas is that in the presence of an obstacle, the field is unchanged if one removes the obstacle and replaces it by appropriate fictitious sources. The field is therefore the sum of a direct field (the field that would be without the obstacle) and the field produced by fictitious sources.

Let Ω be a domain whose regular boundary is Γ and whose set of diffracting points is Δ. We admit that Ω may contain volume sources ρ. By virtue of Huygens' principle, we also put surface sources σ on Γ and diffraction sources λ_i on Δ (Fig. 1). The *energy density* W at any point $\mathbf{x}$ and time t is given by (assuming incoherence),

$$W(\mathbf{x},t) = \int_{\Omega} \frac{\rho\left(\mathbf{y},t'\right)}{4\pi cR^2} d\Omega_{\mathbf{y}} + \int_{\Gamma} \frac{\sigma\left(\mathbf{y},\mathbf{v},t'\right)}{4\pi cR^2} d\Gamma_{\mathbf{y}} + \sum_{i\in\Delta} \frac{\lambda_i\left(\mathbf{v},t'\right)}{4\pi cR^2} \tag{1}$$

where $R = |\mathbf{x}-\mathbf{y}|$ is the source–receiver distance, $\mathbf{v} = (\mathbf{x}-\mathbf{y})/R$ the source–receiver direction, $t' = t - R/c$ the retarded time and c the sound speed (Fig. 1). The domain sources ρ have been assumed omnidirectional but σ and λ_i depend on the emission direction $\mathbf{v}$.

Inside the domain, we may also introduce the *specific radiative intensity* $I(\mathbf{x},\mathbf{v},t)$ defined as the energy flow per unit solid angle and unit area normal to the rays. While the energy density W only depends on the position $\mathbf{x}$ and the time t, the specific intensity depends in addition on the direction vector $\mathbf{v}$. Considering a position $\mathbf{x}$ and an infinitesimal solid angle $d\omega$ about a direction $\mathbf{v}$, the specific intensity is,

$$I(\mathbf{x},\mathbf{v},t)d\omega = \int_{d\omega} \frac{\rho\left(\mathbf{y},t'\right)}{4\pi R^2} d\Omega_{\mathbf{y}} + \frac{\sigma\left(\mathbf{y},\mathbf{v},t'\right)}{4\pi R^2} d\Gamma + \sum_{i} \frac{\lambda_i\left(\mathbf{v},t'\right)}{4\pi R^2} \tag{2}$$