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Shear wave propagation in periodic phononic/photonic piezoelectric medium

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ABSTRACT

Coupled electro-elastic SH waves propagating oblique to the lamination of a one dimensional piezoelectric periodic structure are considered in the framework of the full system of Maxwell's electrodynamic equations. The dispersion equation has been obtained and numerical analyses carried out for two kinds of composites both consisting of two different piezoelectric materials. The results demonstrate the significant effect of piezoelectricity on the widths of band gaps at acoustic frequencies and confirm that it does not affect the band gaps at optical frequencies.

1. Introduction

Photonic crystals are periodic structures consisting of materials with different elastic properties. Many smart structures such as piezoelectric or piezomagnetic composites are made up of two or more different constituents periodically arranged. Compared with the purely elastic crystals they exhibit electric or magnetic effects and thus new acoustic properties. The investigation of acoustic waves in piezoelectric phononic crystals has recently attracted much attention. Hou et al. [1] investigated the elastic band gap structure of a two-dimensional phononic crystal containing piezoelectric material and analyzed the effects of piezoelectricity on the band gaps. Other problems concerning the propagation of acoustic waves in both two and three dimensional piezoelectric periodic structures are considered in [2–14]. In particular, Qian et al. [15] studied the dispersion relations for SH-wave propagation in a periodic layered piezoelectric structure with elastic inclusions for the cases of wave propagation in the directions normal or tangential to the interfaces.

In most of these studies, the quasi-static approximation is adopted for the electromagnetic field. Under this assumption, both the optical effect and the effect from the rotational part of the electric field are neglected. Although it is believed that the optical effect is not significant it might be useful in some applications to accurately predict the piezoelectricity induced electromagnetic radiation. Such applications can for example include optical detection or nondestructive evaluation [16,17].

The problem of electromagneto-acoustic surface waves in a piezoelectric non-periodic medium in a dynamic setting is considered in [18], where the exact solution for the fully coupled SH electromagneto-acoustic surface wave is obtained in a simple closed form. The propagation of electromagnetic and elastic waves in a periodic piezoelectric structure for partial interfacial boundary conditions is discussed in [19], where the interfacial effects are investigated within piezoelectric phononic crystals. Electromagnetic and elastic waves in piezoelectric–piezomagnetic superlattice are investigated in [20].

The purpose of this paper is to investigate the propagation of SH electromagneto-acoustic waves in a one dimensional piezoelectric phononic crystal in a dynamic setting for full contact interface boundary conditions. Taking into account

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both optical effects and the contribution from the rotational part of electric field the solutions obtained can be valid for any wave speed range. They will also provide accurate formulae for acousto-optic interaction in piezoelectric phononic crystals.

2. Statement of the problem

We consider the prorogation of electro-magneto-elastic coupled SH wave in a one dimensional infinite periodic piezoelectric hexagonal media. The problem will be considered in the framework of the full system of Maxwell's equations (non quasi stationary) which will give an opportunity to study the wave dispersion equation in both acoustic and optic wave frequency regions. The interconnected elastic and electro-magnetic excitations in a transversely isotropic piezoelectric crystal with crystallographic axes directed along the OZ direction are described by the following equations and constitutive relations [21]

$$\frac{\partial \sigma_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{1}$$

$$\operatorname{rot} \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \qquad \operatorname{rot} \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{2}$$

$$\sigma_{xx} = c_{11}s_{xx} + c_{12}s_{yy} + c_{13}s_{zz} - e_{13}E_z, \qquad \sigma_{yy} = c_{12}s_{xx} + c_{11}s_{yy} + c_{13}s_{zz} - e_{13}E_z, \tag{3}$$

$$\sigma_{zz} = c_{13}s_{xx} + c_{13}s_{yy} + c_{33}s_{zz} - e_{33}E_z, \tag{4}$$

$$\sigma_{xy} = (c_{11} - c_{12})s_{xy}, \qquad \sigma_{yz} = 2c_{44}s_{yz} - s_{15}E_y, \qquad \sigma_{xz} = 2c_{44}s_{xz} - e_{15}E_x,$$

$$D_x = 2e_{15}s_{xz} + \varepsilon_{11}E_x, \qquad D_y = 2e_{15}s_{yz} + \varepsilon_{11}E_y, \qquad D_z = e_{13}(s_{xx} + s_{yy}) + e_{33}s_{zz} + \varepsilon_{33}E_z,$$

$$B_x = \mu_{11}H_x, \qquad B_y = \mu_{11}H_y, \qquad B_z = \mu_{33}H_z, \qquad s_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right), \tag{5}$$

where u_i are the components of the displacement vector, σ_{ik} and s_{ik} the stress and strain tensors, D_k and E_k the electric displacement and electric field intensity, and H_k and B_k the magnetic field intensity and magnetic induction. The mass density ρ, the elastic, piezoelectric, dielectric and magnetic constants c_{ik}, e_{ik}, ε_{ik} and μ_{ik} are assumed to be periodic functions with respect to x.

In the case of a two dimensional problem (when $\partial/\partial z = 0$) equations and relations (1)–(5) separate into plane and anti-plane problems. The plane problem is with respect to u_x, u_y, E_z, H_x, H_y and is described by the following equations and relations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2}, \qquad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2}, \tag{6}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_{11}}\frac{\partial E_z}{\partial x}\right) + \frac{1}{\mu_{11}}\frac{\partial^2 E_z}{\partial y^2} - \varepsilon_{33}\frac{\partial^2 E_z}{\partial t^2} = e_{13}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right), \tag{7}$$

$$\mu_{11}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0, \qquad \mu_{11}\frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0, \tag{8}$$

$$\sigma_{xx} = c_{11}\frac{\partial u_x}{\partial x} + c_{12}\frac{\partial u_y}{\partial y} - e_{13}E_z, \qquad \sigma_{yy} = c_{11}\frac{\partial u_y}{\partial y} + c_{12}\frac{\partial u_x}{\partial x} - e_{13}E_z,$$

$$\sigma_{xy} = \frac{c_{11} - c_{12}}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right). \tag{9}$$

The following equations and relations describe the anti-plane problem with respect to u_z, E_x, E_y, H_z

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial y} = \rho \frac{\partial^2 u_z}{\partial t^2}, \tag{10}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu_{33}\frac{\partial H_z}{\partial t}, \tag{11}$$

$$\frac{\partial H_z}{\partial y} = \frac{\partial D_x}{\partial t}, \qquad -\frac{\partial H_z}{\partial x} = \frac{\partial D_y}{\partial t}, \tag{12}$$

$$D_x = e_{15}\frac{\partial u_z}{\partial x} + \varepsilon_{11}E_x, \qquad D_y = e_{15}\frac{\partial u_z}{\partial y} + \varepsilon_{11}E_y. \tag{13}$$

$$\sigma_{xz} = c_{44}\frac{\partial u_z}{\partial x} - e_{15}E_x, \qquad \sigma_{yz} = c_{44}\frac{\partial u_z}{\partial y} - e_{15}E_y. \tag{14}$$