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Fully dispersive dynamic models for surface water waves above varying bottom, Part 2: Hybrid spatial-spectral implementations

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ABSTRACT

In Part 1 (van Groesen and Andonowati [1]), we derived models for the propagation of coastal waves from deep parts in the ocean to shallow parts near the coast. In this paper, we will describe hybrid spatial-spectral implementations of the models that retain the basic variational formulation of irrotational surface waves that underlays the derivation of the continuous models. It will be shown that the numerical codes are robust and efficient from results of simulations of two test cases of waves above a 1:20 sloping bottom from 30 m to 15 m depth: one simulation of a bichromatic wave train, and one of irregular waves of JONSWAP type. Measurements of scaled experiments at MARIN hydrodynamic laboratory and simulations with two other numerical codes will be used to test the performance. At the end of the full time trace of 3.5 h details of the irregular waves that travelled over more than 5000 m are clearly resolved with a correlation of more than 90%, in calculation times of less than 5% of the physical time. Also freak-like waves that appear in the irregular wave are shown to be modelled to a high degree of accuracy.

1. Introduction

This paper will show the possibility to accurately simulate 1D waves propagating from deep water to the shallower coastal area. Such simulations can support various commercial–industrial offshore activities in the coastal area such as oil and gas exploration, and can investigate environmental issues for a sustainable coastal development.

We show the applicability of wave models that are derived in Part 1 [1] for coastal wave simulations and quantify the accuracy by comparing results of simulations with measurements performed at MARIN, the Maritime Research Institute Netherlands, Wageningen. A relatively simple implementation of the model equations will lead to efficient simulations that can capture the measured wave distortions above a sloping bottom to a good degree of accuracy. Two different types of waves are considered that will run over a 1:20 slope from a depth of 30 m to a depth of 15 m: a rather low amplitude bichromatic wave, and a higher irregular wave with significant wave height of approximately 3 m.

A summary of the basic model equations that were derived in Part 1 is given in Section 2. In Section 3, the numerical implementation of the model equations is described in detail, with various simplifications that have shown not to affect the quality of the simulations in a systematic way. In Section 4, we describe the experimental layout of the geophysical domain and the peculiarities of the wave types that were generated and measured. Numerical parameter settings are provided and we show and comment on the results of simulations by comparison with measurements and with results of two other codes, the commercial software MIKE 21 BW, and an Optimized Variational Boussinesq code [2]. In this Section 4, we also show in detail the quality of simulation of large amplitude, freak-like waves that appear in the irregular waves; the results indicate

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that the present code is capable to identify the appearance of such waves from the numerical simulations. Conclusions and remarks are formulated in the final Section 5.

2. Summary of the model equations

In this section we will describe briefly the basic models that were derived in Part 1 [1]. To that end we repeat that the basic problem of surface fluid waves is to translate the interior fluid motion, which is described by the Laplace problem for the internal velocity potential, to the surface dynamic equations. Specifically, the Dirichlet-to-Neumann (DtN) operator that transforms the boundary values of the potential at the water surface into values of its normal velocity is needed. Various methods can be distinguished by the way how this DtN operator is approximated.

Fully numerical methods calculate at each instant the interior Laplace problem and transfer the information to the dynamic surface equations to advance the surface elevation in time. Instead of that, one can avoid the numerical calculation of the interior fluid motion by approximating its solution with an underlying model to approximate the DtN operator and implement this directly in the surface equations that can then be advanced in time in a numerical way. This then leads to Boussinesq-type of equations, with the Korteweg–de Vries and the classical Boussinesq equations as prime examples. The underlying model commonly uses series expansions with assumptions about the ratio between wavelength and wave amplitude to be specified. Improvements of the resulting dispersion relation to waves that do not satisfy the assumed relation are commonly needed to extend the applicability. The well-known MIKE 21BW software is an example, based on high-order dispersion improvements; see for instance [3,4].

The variational theory of surface waves that is pursued in this paper is based on Luke's variational principle [5] and Zakharov's Hamiltonian formulation [6], as described in Part 1. Then the dynamic equations at the free surface are directly expressed in terms of the surface potential and surface elevation through variational derivatives of the Hamiltonian, thereby giving a dimension reduction as in Boussinesq models. Within the structure of the Hamilton equations, various papers, initiated by Craig and Sulem [7], see [8,9], use a convergent Taylor approximation of the DtN operator. Different from this method, in [10] the surface elevation and the potential at the still-water level are used as primary variables. The Hamiltonian structure is retained by performing this (non-canonical) transformation in the underlying action principle. Then the DtN operator is explicit, and approximations of the Hamiltonian were derived using Taylor expansions around the still water level for which the Hamiltonian is exactly known when the bottom is flat. Using spectral methods, the pseudo-differential operators can be handled as multiplications in Fourier space, and dispersive properties are exact for infinitesimal waves of any wavelength. For varying bottom, the quasi-homogeneous approach is applied to operators in the Hamiltonian, which leads to consistent symmetries for operators in the resulting Hamiltonian equations, which would not be the case if this procedure is applied directly in the Hamilton equations.

In Part 1 we derived successively more complete models based on the so-called AB-equation in [10] for uni-directional waves above flat bottom, extended to the case of varying bottom in the linear and the quadratic terms. Upon including also a linear coupling with reflected waves, a nonlinear set of bi-directional equations with good dispersive properties was derived. If these equations are restricted to harmonic waves or narrow-band wave groups, they are a generalization of the mild-slope equations that are known in the literature. The generalization consist of an extension to waves with broad spectrum, and with second order nonlinear terms. We now summarize the model equations to be used for the simulations in this paper.

Starting point is the AB-model for nonlinear uni-directional waves above flat bottom, given by the following equation for the surface elevation $\eta = \eta(x, t)$ [10]

$$\partial_t \eta = -\mathcal{AB}\,(\eta)\,, \tag{1}$$

where $\mathcal{AB}$ is symbolic notation for the following expression

$$\mathcal{AB}\,(\eta) = A\left[\eta + \frac{g}{4}\,(B\eta)^2 + \frac{g}{2}B\,(\eta B\eta) - \frac{1}{4g}\,(A\eta)^2 + \frac{1}{2g}A\,(\eta A\eta)\right]. \tag{2}$$

The operators A and B appearing in this equation will be detailed below.

The model that includes effect of reflections from bottom variations, called the BiAB model, is then given by

$$\begin{cases} \partial_t \eta = -\mathcal{AB}\,(\eta) + \zeta \\ \partial_t \zeta = +\mathcal{AB}\,(\zeta) - \mu\eta. \end{cases} \tag{3}$$

Here ζ can be interpreted as an approximation of the reflected wave, and μ is the reflection coefficient (operator) that will be described below. Since in most cases (including the test cases of this paper) the reflection coefficient is rather small, the linearized equation for ζ will suffice, and the nonlinear effects in the coupling between η and ζ, which is of higher order than the linear coupling, has been neglected:

$$\begin{cases} \partial_t \eta = -\mathcal{AB}\,(\eta) + \zeta \\ \partial_t \zeta = +A\,(\zeta) - \mu\eta. \end{cases} \tag{4}$$

This is the equation that will be used in the simulation of the test cases in this paper. This model will be called the BiAB model to express the bi-directionality.