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Acoustic radiation force acting on absorbing spherical shells

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Abstract

The frequency dependence of the acoustic radiation force function, Y_p, for an absorbing spherical shell suspended in a plane incident sound field is analysed, in relation to its thickness and the content of its hollow region. The theory is modified to include the effect of hysteresis type of absorption of compressional and shear waves in the material. The results of numerical calculations are presented for two absorbent materials, with the hollow region filled in water or air. These results show (i) how damping (due to absorption), (ii) change of the interior fluid in the shell's hollow region and (iii) and the exterior fluid surrounding the shell affect the acoustic radiation force.

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1. Introduction

Acoustic radiation force was probably first investigated by Rayleigh [1]. In a pioneering work, King [2] calculated the radiation force experienced by a rigid sphere in standing and travelling acoustic plane-wave fields. For a sphere whose diameter is much smaller than the sound wavelength, he found that the direction of the radiation force depends on the density ratio of the sphere and surrounding medium. Heavier spheres are attracted to velocity antinodes and lighter spheres to velocity nodes. Moreover, the acoustic radiation force exerted by plane travelling waves is much smaller than that exerted by standing waves of the same amplitude. Later, Yosioka and Kawasima [3] extend King's work to include the sphere compressibility. Soon after, much theoretical and experimental work has been performed on the radiation force exerted by a plane acoustic field on a sphere [2–8].

Subsequently, the theory of the acoustic radiation force on elastic cylindrical and spherical shells has been developed [9] and tested on materials such as aluminum and stainless steel. In that work, it was demonstrated that

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the acoustic radiation force is very sensitive to the longitudinal and shear sound wave velocities and thickness of the shells. The frequency dependence of acoustic radiation force is very relevant of the material mechanical properties and it has series of maxima and minima corresponding to resonance frequencies of shells' elastic vibrations [10]. Thus, some of the material's elastic constants can be determined from the resonance phenomenon. A current application is for transducer calibration since the acoustic radiation pressure exerted by ultrasonic waves on an object provides an accurate and primary method for determining the intensity of incident acoustic waves [8].

Very little is known, however, about the radiation force on absorbing spherical shells placed in a plane incident ultrasound field and the theory as it stands, may not be applicable to absorbing materials such as lead, epoxy and polymers since it does not include effects due to sound absorption of shear and compressional waves in the shell material. The development of the acoustic radiation force theory for absorbing spherical shells is necessary for many medical (drug delivery by radiation force-guided encapsulated polymeric shells [11]) and industrial (non destructive evaluation) applications.

Hasegawa and Watanabe [12] studied the acoustic radiation force exerted on absorbing spheres. In this work, we extend the theory of the acoustic radiation force due to plane progressive waves [9] to include the effect of sound (or ultrasound) absorption inside the spherical shells' material. The sound absorption was linearly dependent on frequency, describing the behavior of many polymeric materials. Numerical calculations are carried out with emphasis on the effects of the shells' thickness in a large range of frequency. These results are also compared with the calculations for the case where the material is considered as non-absorbent and in the case where the fluid outside and inside the shell is not the same. The case of viscoelastic shells filled with water and immersed in a high density fluid is also examined. This study shows that the theory developed here is more general and contains the results of the work done by Hasegawa and Watanabe [12].

2. Scattering from an absorbing spherical shell

The axis of the spherical shell of outer radius a, inner radius b, is taken to coincide with the z-axis of the coordinate system. The outside and inside fluid mass densities are ρ and ρ', respectively, and the shell's mass density is denoted by ρ^*.

In the spherical coordinates, the incident wave can be represented by:

$$P_i = P_0 \sum_{n=0}^{\infty} (2n+1)(-i)^n j_n(kr) P_n(\cos\theta), \tag{1}$$

where P_0 is the amplitude, $j_n(x)$ the spherical Bessel function of the first kind of order n and argument x, k the wave number in the fluid medium and $P_n(\cdot)$ are the Legendre polynomials. For convenience, the time factor $\exp(i\omega t)$ is not explicitly written in Eq. (1).The scattered pressure takes the form:

$$P_\mathrm{s} = P_0 \sum_{n=0}^{\infty} (2n+1)(-i)^n d_n h_n^{(2)}(kr) P_n(\cos\theta), \tag{2}$$

where $h_n^{(2)}$ is the spherical Hankel function of the second kind and d_n are the shell scattering coefficients to be determined from the boundary conditions and are given by:

$$d_n = -\left[\frac{F_n j_n(x) - x j_n'(x)}{F_n h_n^{(2)}(x) - x h_n^{(2)'}(x)}\right]. \tag{3}$$