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Finite-amplitude shear wave in pre-stressed thin elastomers

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Abstract

We examine the elastic shear waves generated in a thin pre-stressed elastomer layer that is sandwiched between two relatively thick steel plates and is subjected to an elastic shear wave traveling in one of the steel plates. The elastomer layer has been deformed in uni-axial strain in advance, producing in the layer very large axial and lateral compressive stresses of the order of its bulk modulus. Deformations of this kind are produced in the thin layer when the sandwich structure is impacted by another steel plate at an oblique angle. Our results are thus relevant to the analysis of such pressure-shear plate impact experiments.

Keywords: Finite-amplitude shear waves; Pre-stressed elastomers

1. Introduction

Most polymers are elastomeric at temperatures above their glassy transition, T_g. They are solids with very small shear moduli at ordinary pressures and temperatures, but have relatively large bulk moduli. Commercially available polyurethane, for example, is reported by Mott et al. [1] to have a shear modulus of about 6 MPa and a bulk modulus of 2.4 GPa. Similarly, polyurea which is closely related to polyurethane, has a very small shear modulus at room temperature and ordinary pressures, but a relatively large bulk modulus. Elastomers of this kind are *soft solids*, strong in tension and resistant to abrasion and impact. They are commonly used for metal and concrete surface protection. Solids of this kind are fluid-like when they are impacted by high-velocity hard projectiles, but retain their solid-like integrity and regain their initial shape upon the completion of the impact event.

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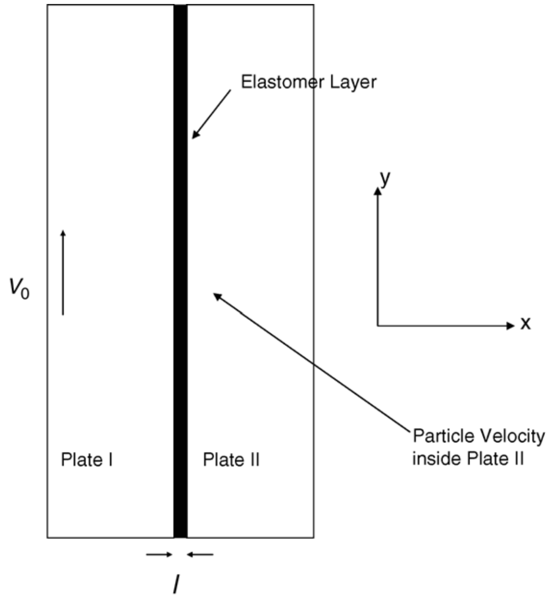

Fig. 1. A thin elastomer layer is sandwiched between two thick steel plates and uni-axially pre-strained. A shear wave with particle velocity V_0 traveling through plate I in the x-direction, impacts the elastomer. The shear velocity at a typical point inside the rear plate is calculated based on the properties of the elastomer.

The shear moduli of elastomers are pressure-dependent, and can reach and exceed 1 GPa when the material is suitably pre-compressed. Confined compression Hopkinson bar and pressure-shear plate impact experiments have been performed to evaluate the pressure-dependence of the shear modulus of elastomers [2,3]. Here, we confine attention to the analysis of shear-wave propagation in a thin layer of an elastomer that is sandwiched between two relatively thick steel plates (Fig. 1), and is pre-deformed in uni-axial strain, producing very large axial and lateral compressive stresses in the thin elastomer.

The problem of finite deformation elastic waves has been studied extensively in the literature, especially for hyper-elastic materials. Truesdell [4] has studied the propagation of small oscillation superposed on large deformations in hyper-elastic materials based on the work by Toupin and Bernstein [5]. Bland [6] has addressed the problem of *simple* waves for which the propagation direction coincides with one of the principal axes of the stress or deformation tensors. Chu [7] has discussed wave propagation in incompressible materials. Truesdell [8,9] has introduced the concept of hypo-elasticity and shortly afterwards Noll [10] has shown that it encompasses the Cauchy elasticity. Bernstein [11] found the necessary and sufficient conditions for a hypo-elastic material to be elastic in the sense of Cauchy and even furthermore in the sense of Green (hyper-elasticity). It is now well established that this general framework is a natural choice for most numerical routines and can be extended to plasticity and other material properties [12]. Truesdell [8,9,13] has solved a few simple problems using this model. More recently, Lin et al. [14] have studied uniform quasi-static deformation in a closed path, which includes uniform simple shearing, for several hypo-elastic material models.

In the present work, we use a hypo-elastic model based on a linear relation between the Jaumann rate of the Cauchy stress and deformation rate tensors. As is known, models based on other objective stress rates and material strain rates can be rendered equivalent by proper selection of the tangential moduli; for a discussion of this equivalence see Nemat-Nasser [15].