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SCIENCE DIRECT
Wave Motion 43 (2005) 51–66
WAVE MOTION
www.elsevier.com/locate/wavemoti

Moving oscillating loads in 2D anisotropic elastic medium: Plane waves and fundamental solutions

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Received 25 November 2003; received in revised form 18 May 2005; accepted 9 June 2005

Abstract

The paper considers a concentrated point force moving with constant velocity and oscillating with constant frequency in an unbounded homogeneous anisotropic elastic 2D medium. Such problems come from the problems of a source that acts along the line in the corresponding three-dimensional anisotropic medium. Fundamental solutions for two-dimensional problems with moving oscillating sources lay the foundation for constructing solutions for more complicated problems and applying the boundary integral equation method.

The properties of plane waves, their phase, slowness and ray or group velocity curves are determined in a moving coordinate system. The use of Fourier integral transform techniques and the properties of plane waves enables to obtain an explicit representation for elastodynamic Green's tensor for all types of the source motion as a sum of integrals over a finite interval. The quasistatic and dynamic components of the Green's tensor are obtained.

The stationary phase method is employed to derive an asymptotic approximation of the far wave field. Simple formulae for the Poynting energy flux vectors for moving and stationary observers are also presented. It is noted that in far zones the wave fields are subdivided into separate cylindrical waves under kinematics and energy.

It is shown that motion brings some difference in the far field properties, exemplified by the modification of the wave propagation zones and the change in their number, emergence of fast and slow waves under trans- and superseismic motion and etc.

Keywords: Fundamental solutions; Plane waves; Anisotropic elastic medium

1. Introduction

The theory of elasticity with moving source has a range of important applications. They are associated with high-speed transport design and necessity to evaluate the influence of elastic waves from moving objects on different

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doi:10.1016/j.wavemoti.2005.06.002

constructions. Theoretical aspects of these problems are also of interest. The changes in the character of mechanical fields considerably depend on the behavior of source motion and are caused by the type of differential equations ranging from elliptical to hyperbolic ones.

Among the steady-state problems with moving sources the problems with moving and oscillating sources are the most complicated ones. We shall use the following terminology. If the source moves with constant velocity $\mathbf{w}$ and simultaneously oscillates with the frequency ω, then we consider problem B, but in the case $\mathbf{w} = 0$ we have a classical harmonic problem with oscillating source and refer it to as problem A.

Nowadays considerable progress is made in the investigation of problem B. The correspondence principles between problems B and A, analogous to those in fluid mechanics [1], are stated in [2]. The principles for unique solution selection are investigated in [2,3]. The energy principles and general theorems about energy transport are formulated in [3], several problems B for isotropic media are solved in [4–7].

There is a limited number of papers devoted to particular problems B for anisotropic elastic media. Plane waves and fundamental solutions for problem B for three-dimensional anisotropic media are studied in [8,9]. Antiplane problems are considered in [9], and approaches to problem B for anisotropic elastic and piezoelectric waveguides are suggested in [10].

For anisotropic media, even in problem A, fundamental solutions have a rather cumbersome form and can be presented only in the integral form. The papers [11–13] made considerable contribution to the development of methods for solving dynamic problems for anisotropic media. Search of new forms of fundamental solutions for problems A for anisotropic media was made further in many papers, for instance in [14–18]. It is noted that new specific anisotropy appears for problems B in a moving coordinate system. This anisotropy is due to the source motion and additionally makes the form of fundamental solutions more complicated.

In the present paper we investigate the properties of plane waves and fundamental solutions of problem B for anisotropic elastic plane.

It is well-known that for many crystallographic classes of anisotropic materials it is possible to formulate problems A and B in conditions of plane deformation. For existence of a plane problem several elastic modules must be equal to zero. Thus, for plane deformation in the plane $0x_1x_3$ for elastic modules in two-index notation we require

$$C_{14} = C_{34} = C_{45} = C_{16} = C_{36} = C_{56} = 0,$$

for plane deformation in the plane $0x_1x_2$

$$C_{14} = C_{24} = C_{46} = C_{15} = C_{25} = C_{56} = 0,$$

and for plane deformation in the plane $0x_2x_3$

$$C_{25} = C_{26} = C_{35} = C_{36} = C_{45} = C_{46} = 0.$$

The vectors of the external force in the plane deformation problems should not have non-zero components in perpendicular direction and should not depend on the perpendicular to the plane coordinate.

We assume that all of the above-mentioned conditions are implemented. To be more specific, let us accept that plane deformation takes place in the plane $0x_1x_2$, and therefore the mechanic displacement vector has non-zero components u_1 and u_2: $\mathbf{u} = \{u_1(\mathbf{x}, t), u_2(\mathbf{x}, t)\}$, $\mathbf{x} = \{x_1, x_2\}$. For harmonic vibrations with the frequency ω the solution is given in the form

$$\mathbf{u} = \mathbf{v} \exp(\mathrm{i}\omega t). \tag{1.1}$$

In the problem with moving sources we consider two coordinate systems. Let $\{\xi_1, \xi_2\}$ be a fixed coordinate system with time τ, and let $\{x_1, x_2\}$ be a coordinate system, which moves relative to the fixed system with constant