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RESEARCH PAPER

Stochastic Bottleneck Model with Heterogeneous Travelers

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Abstract: This paper studies commuters' departure time choice behavior along a bottleneck constrained highway connecting a resident area and a workplace. Commuters are differentiated by their value of time, and the bottleneck capacity is assumed to follow a uniform distribution within a range. The equilibrium properties of the proposed model are derived and individual travel cost is obtained. It is found that at equilibrium, the increase of capacity variability would change the commuters' travel behavior by increasing the expected equivalent travel time and lengthening the peak period. Every commuter is better off by reducing the individual travel cost if the stochasticity of the capacity decrease, and commuters with high value of time benefit more than those with low values. Numerical results are presented to support analytic findings.

Key words: highway transportation; travel behavior; bottleneck model; stochastic capacity; heterogeneity

1 Introduction

The well known bottleneck model was originally developed by Vickery[1], which formulates an endogenous departure time choice of commuters and leads to an equilibrium costs for all commuters. This model considers a situation where a fixed number of identical commuters travel from home to work place along a single road during the morning peaks. There is a single bottleneck on the road with a fixed capacity or service rate, and if the arrival rate at the bottleneck exceeds this capacity, a queue will develop. The distribution of arrival time is such that it is physically impossible for all commuters to arrive at work exactly on time and don't wait in line; consequently, to minimize the trip cost, commuters have to obtain a trade-off between the travel time cost relating to the queue length and the schedule delay cost of arrival early or late. Equilibrium obtains when no individual has an incentive to alter his/her departure time.

Bottleneck model provides a simple and direct way to gain qualitative insights for morning commuting behavior, and it has been extended in various ways[2–4]. Mahmassani and Herman[5] considered simultaneous departure time and route choices in morning commute problem, and Kuwahara[6] studied the problem under a network with two origins, one destination, and connected by consecutive bottlenecks. Tabuchi[2–4] investigated a competitive transportation system which contains the physically separated transit and highway modes, and concluded that the road pricing can be regarded as a measure for restraining auto use and providing revenue for mass transport improvement. Huang[8,9] introduced the concept of body congestion in Tabuchi's work and stressed the heterogeneity of commuters. In reality, not only does the travel time increases with the traffic volume, but there is a wide range of randomness in micro behaviors and traffic condition. Variations occur for unreliability of travel time in weather conditions, accidents, road maintenance, etc. In recent, increasing researchers have turned their attention to the importance of uncertainty in travel behavior[10,11].

These studies lead to a better understanding of the traveling patterns based on the bottleneck model, however, the majority of these studies made an assumption that travelers are homogeneous, i.e., all the travelers have the same preference for arriving early/late and have the identical value of time (VOT). In real circumstances, commuters are heterogeneous in nature due to the income level and scheduling preferences. Clearly, this assumption can significantly influence the results, particularly the travel patterns and the social welfare.

In general transportation equilibrium problems, heterogeneity is addressed by introducing groups that have different travel cost functions. In the static context, some scholars gave the general formulations of multi-class user

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equilibrium problems (Dafermos[12]; Smith[13]). A typical UE problem with multi-class travelers can be formulated as a variational inequality (VI) (Yang and Huang[14]; Nagurney[15]), and may be reduced to a single-group problem by constructing a new network with k copies of the original network (Dafermos[12]; Nagurney[15]). de Palma and Lindsey[16] developed a bi-level model with several toll schemes to find the effect of heterogeneity on the social welfare.

Dynamic traffic models usually assume that a driver faces a schedule delay if he/she does not arrive at his /her preferred arrival time. In the conventional linear specification, the cost per hour of earlier arrival than preferred time is β, the cost per hour of later arrival is γ, the value of time is α. A number of studies have investigated the effect of heterogeneity on these parameters. Cohen[3] examined the fine tolling with two user groups different in their preferred arrival time and value of time. de Palma and Lindsey[17] made a comparison between the bottleneck model with homogeneity and that with heterogeneity. This paper investigates the morning commuting problem which is formulated as a bottleneck model with stochastic capacity. We derive the model's analytical solution and investigate the properties of the model with consideration of heterogeneous travelers.

2 Classic bottleneck model

In this study, we take a highway connecting a residential district (H) with a central business district (W). There is a single bottleneck located at the end of the highway, with a deterministic capacity. During the morning peaks, each individual travels by his/her own car along the single road and wants to arrive at the work place on time. Because of the limitation of the bottleneck capacity, commuters may arrive at work early or late and thus endure the penalty of the schedule delay. Without loss of generality, the cost of commuters consists of two components: the costs of travel time and the cost of schedule delay. Given this, each traveler chooses an optimal departure time so as to minimize his/her trip cost. The trip cost for travelers departing at time t can be formulated as follows:

$$c(t) = \alpha T(t) + \beta \cdot SDE(t) + \gamma \cdot SDL(t) \quad (1)$$

where, α, β and γ denotes the values of travel time, schedule delay early (SDE) and schedule delay late (SDL), respectively. Let s be the capacity of the bottleneck, and $T(t) = T_0 + Q(T_0 + t)/s$ be the travel time of departing at time t, T_0 denotes the free flow travel time and we set $T_0 = 0$ for simplicity, and $Q(t)$ denotes the length of the queue behind the bottleneck at time t. With this definition, the cumulative departures can be written as follows:

$$R(t) = \int_{t_e}^{t} r(\omega)\mathrm{d}\omega \quad (2)$$

where, t_e is the earliest time, $r(t)$ is the departure rate. The queue length behind the bottleneck is

$$Q(t) = \max\left\{R(t) - s\left(t - t_e\right), 0\right\} \quad (3)$$

The equilibrium condition for commuters' departure time choice in a single bottleneck is: no commuter can reduce his/her travel cost by unilaterally altering his/her departure time. This implies that the commuters' travel cost is fixed according to the time instant for the positive departure rate, i.e., $\mathrm{d}c(t)/\mathrm{d}t = 0$ if $r(t) > 0$. Under this condition, we can obtain the first and the last departure time of the peak hour, the departure rate, the mean queue length and the individual's mean trip cost.

3 Model description

3.1 Heterogeneous travelers

In the real world, the income level of an individual largely determines how much the individual's value of time (VOT). In this context, we assume that each commuter has a different value of time (VOT) and follows a probability distribution:

$$F(\omega) = \Pr\left\{\alpha \le \omega\right\} \quad (4)$$

Moreover, the flexibility of one's work schedule is determined by the value of α/β and α/γ. For simplicity, we assume the relative value of queuing and schedule delay is the same, i.e., $\beta/\alpha = \eta_1$, $\gamma/\alpha = \eta_2$. It means that commuters with higher VOT will encounter the larger schedule delay cost. Clearly, under the previous assumption, the parameter β and γ should follow the same probability distribution as α. Let the commuters is ordered in the decreasing order of their values of schedule delay, then we can obtain a function $\alpha(x)$, representing the value of time of the xth commuter. The relationship between $\alpha(x)$ and the cumulative distribution function of VOT can be formulated as follows:

$$\alpha(x) = F^{-1}\left(\frac{x}{N}\right) \quad (5)$$

We can also obtain the relationship between $\alpha'(x)$ and the VOT's probability density function, $f(\alpha)$ as follows:

$$\alpha'(x) = \frac{1}{Nf(\alpha(x))} \quad (6)$$

Eq. (5) and Eq. (6) provide a convenient way to derive $\alpha(x)$ from the probability density or cumulative density function of VOT.

3.2 Stochastic capacity

In deterministic settings, the bottleneck capacity is always assumed to be fixed. However, in reality, the capacity can be stochastic due to bad weather conditions, accidents, road maintenances, etc. The capacity fluctuation leads to variability of travel time and trip cost, which in turn directly influences the commuters' departure time choice behaviors. To describe the uncertainty of the capacity, we assume the bottleneck capacity is a stochastic variable and follows a uniform distribution with interval $[\theta\overline{s}, \overline{s}]$, where $\overline{s}$ is the design capacity and $\theta(\theta < 1)$ is a positive parameter which denotes