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RESEARCH PAPER

# Stochastic Traffic Equilibrium Based on Travel Time Robust Reliability

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**Abstract:** An assumption that pervades the current reliable traffic equilibrium problem is that probability distributions of the origin-destination (O-D) demand or/and link capacities are known explicitly. However, these distributions are difficult to be accurately obtained. This paper relaxes this assumption. It only needs to know the first m moments of travel demand (where m is a positive integer associated with the formulation of link cost function), and then applies two worse-case Value-at-Risk (WVaR) and Conditional value-at-risk (CVaR) risk measures to define robust percentile travel time (RPTT) and robust mean-excess travel time (RMETT) and prove that this two kinds travel time is equal under general distribution. By incorporating the defined travel time and travelers' perception error, the robust percentile stochastic user equilibrium (RPSUE) or robust mean-excess stochastic traffic equilibrium model (RMESTE) is proposed, which is formulated as an equivalent route-based variational inequality. Conditions are established guaranteeing existence of this equilibrium. A heuristic solution problem is introduced to solve the variational inequalities problem. A numerical example is used to illustrate the applications of the proposed model and the solution algorithm.

**Key Words:** system engineering; stochastic traffic equilibrium problem; worst-case value-at-risk; worst-case conditional value-at-risk; robust percentile travel time; robust mean-excess travel time; variational inequalities

## 1 Introduction

The traffic equilibrium problem is the most fundamental and critical issue in transportation research. Given the origin-destination (O-D) demand and travel cost function for each link of the transportation network, the traffic equilibrium problem determines the traffic flow distribution and various performance measures of the network according to certain route choice criteria. The traditional traffic equilibrium problem assumes the O-D demand or/and link capacity of the network are determined. However, the realistic traffic system is affected by various uncertain factors, which roughly can be classified into supply and demand sides. For example, the travels' activity pattern contributes to uncertain O-D demand, while the adverse weather, the road accidents, or traffic management and control lead to a degradation in link capacity. The impact of uncertain O-D demand or/and link capacity on the traveler occurs through the induced travel time variability. A recent empirical study[1] revealed that travel time variability is viewed as a risk in route choice decision. Travelers are interested in not only travel time saving but also risk reduction when making their route choice. The traditional traffic equilibrium model adopts the expected travel time or expected perceived travel time as a criterion for making a route choice, which obviously cannot really reflect the traveler's route choice decision process. In order to accurately present the traveler's route choice behavior, some risk measures in the finance field, such as Value-at-Risk (VaR)[2] and Conditional Value-at-Risk(CVaR)[3], are adopted as the criterion for making a route choice. Value-at-Risk (VaR) is a measure that estimates the maximum potential loss at a certain probability level in the financial field, that is, it provides information on the amount of losses that will not be exceeded with a certain probability. Mathematically, VaR is defined as a percentile-of-a-loss distribution. In transportation, it is regarded as percentile route travel time (PTT) for a desired on-time arrival probability, which represents the reliability aspect of travel time variability. Nie[4] presented a multi-class percentile user equilibrium (PUE) model under a stochastic capacity. The stochastic link capacity is incorporated by

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modeling the service flow rate of each road segment as a random variable. The probability density functions of road capacity are specified exogenously with flow-dependent parameters. The distribution of route travel time is obtained by directly convoluting the link travel time distribution. Hall[5] observed that travelers tend to reserve a safety margin to hedge against for the variation of travel time. Such a behavioral tendency leads to the concept of effective travel time (ETT) or the concept of travel time budget (TTB), which is defined as the sum of the expected travel time and this safety margin. The safety margin is the product of the standard deviation of travel time and a scalar called punctuality parameter, which reflects the traveler's attitude toward the risk. In fact, if travel time is normally distributed, then the punctual parameter has a one-to-one correspondence with the on-time arrival probability, in which case, the ETT or TTB is equal to PTT. However, this equivalence does not hold in general[6].

Recent studies used the notation of ETT or TTB to model network uncertainty in the traffic equilibrium model. Lo *et al*.[7] applied the concept of TTB to propose a probabilistic user equilibrium (PUE) model, in which the link capacity is assumed to be uniform distribution and the O-D demand is fixed. Shao *et al*.[8] presented a demand-driven travel time reliability-based user equilibrium (DRUE) model based on the same concept. Shao *et al*.[9] further extended the DRUE model into the reliability-based stochastic user equilibrium (RSUE) by incorporating the perception error. Lam *et al*.[10] extended the RSUE model by accounting for the impacts of adverse weather conditions on a road network with uncertainties in demand and supply. Moreover, Siu and Lo[11] also proposed a mixed-user equilibrium model under doubly uncertain transportation networks with stochastic link degradation and stochastic demand based on the TTB. Chen *et al*.[12] indicated that the travel time variability can be represented by two different aspects: reliability aspect and unreliability aspect. All the equilibrium models just mentioned represent the travel time variability from the reliability aspect. Walting[13] developed a late-arrival penalize user equilibrium (LAPUE) model by incorporating a schedule delay term into disutility function to penalize late arrival for a fixed departure time accounting for unreliability aspect of travel time variability.

To adequately present the traveler's route choice decision process under travel time variability, another risk measure, that is, CVaR, is adopted as the criterion for making a route choice. CVaR is defined as the mean of the tail distribution exceeding VaR. In transportation, it is described as the conditional expectation of the travel time exceeding the corresponding percentile travel time. Chen *et al*.[14,15] applied the CVaR risk measure in an attempt to define the mean-excess travel time (METT), proposed an alpha-reliable mean-excess travel equilibrium (METE) model, and compared the three-user equilibrium models (MTT-based, TTB-based, and METT-based). Chen *et al*.[12] extended the METT model into the stochastic METT model by explicitly modeling the stochastic perception errors within the traveler's route choice decision process. Moreover, Nie and Wu[6] analyzed the relationship between stochastic dominance theory and some reliability-based route choice models, such as PTT, TTB, METT, etc, and provided a unified and computationally viable solution framework that avoids an enumeration of all routes.

The traffic equilibrium models just mentioned supposed the fact that the O-D demand or/and link capacity follow some specified distribution, and then, the derived route travel time follows the normal distribution by using the center limitation theorem, or it is directly assumed that the route travel time follows a normal or multinomial normal distribution. However, in reality, it is generally difficult to know the probability distribution of the O-D demand or/and link capacity. Walting[13] indicated that the route travel time generally does not follow a normal or multinomial normal distribution, and listed the advantages and disadvantages of the assumption that the route travel time follows a normal or multinomial normal distribution. This article relaxes this assumption and presents a stochastic transport equilibrium problem based on robust travel time reliability under distribution-free demand in the sense that the methodology only requires the first m moments of O-D demand; respectively defines robust percentile travel time (RPTT) and robust mean-excess travel time (RMETT) by using worst-case VaR and worst-case CVaR risk measures[16]; and proves that these two kinds of travel time are equal under a general distribution. By means of the defined travel time and travelers' perception, the stochastic user equilibrium model that is based on robust travel time reliability is proposed. The proposed model is formulated as a variational inequalities (VI) problem, and then, we establish the existing condition of this equilibrium. A heuristic algorithm is developed that solves this variational inequities problem. The numerical results demonstrate the validity of the formulation and solution algorithm.

This article is structured as follows. In Section 2, the concepts of RPTT and RMETT in a stochastic network are introduced. Model formulation and solution existence are also provided. In Section 3, a routed-based traffic assignment algorithm based on the gap function is provided, which determines the equilibrium flow pattern. In Section 4, a numerical example is presented that demonstrates the validity of the proposed model and solution algorithm. Finally, conclusions are drawn, and recommendations for future research are given in Section 5.

## 2 Model and formulation

### 2.1 Notations and assumptions

Let us consider a strongly connected network $G(N, A)$, where $N$ and $A$ denote the sets of nodes and links, respectively.

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