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RESEARCH PAPER

Side Constrained Traffic Assignment Model Based on Network Dual Equilibrium

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Abstract: One side constrained traffic assignment model is presented based on the local nearsighted user equilibrium principle by taking advantage of network dual equilibrium theorem. The traffic flow and travel time are regarded as a pair of dual variables. The analysis is started from the basic elements of network. At first, the dual relationships between the conservation conditions of node's flow and the shortest travel time from origin to the node are considered. Then the dual relationships between link flow and travel time constrains on link with nearsighted users are also analyzed. At last, through combining these dual relationships and adding generalized side constraints, new traffic assignment model is built. How to embody two behavioral assumptions (priority of playing cards and elasticity of route adjusting on route) in the proposed algorithm to the model is analyzed. A searching method of effective path set is given by using the model's solution results of unique link flow with respect to origin and destination pair. Several numeral examples are given to prove the validity and efficiency of the model and related algorithm. The results for the side constrained model are explained by congestion pricing and queue delay on links.

Key Words: system engineering; traffic assignment; network equilibrium; side constraints; effective path

1 Introduction

Traffic assignment refers to the process of finding out the pattern of network flow and actual links' traveling cost under the situation that the features of supply and demand of the network have known. During the traffic assignment, the travelers' behaviors of path finding often need to be presumed. The commonly used assumptions are the Wardrop's first and second principles that can be applied to different situations. But the two assumptions have one common presupposition that every traveler knows not only the perfect information of the whole network, but also uses it indifferently to calculate and find the best path for himself. In literature [1], through modifying the unilateral behavior assumption, a generalized Wardrop's principle considering the cooperation in different levels was proposed. In this paper, we will relax the perfect information assumption and assume that travelers choose their travelling paths only knowing the local traffic information of the network. We call this new behavior assumption "Local nearsighted user equilibrium principle".

An effective approach of modeling traffic assignment was adding side constraints. Engineers can build some practical side constraints according to their experiences. These side constraints had a relatively good explanation. For example, they can be explained as queue delay on road or congestion pricing. Some can be used to modify the delay functions of links. In literatures [2] and [3], through introducing flow limitations on links, the congestion characteristic of the network was modeled. The side constraints were used to treat the signal control on road in literatures [4] and [5]. The related researches were summed up in literatures [6] and [7] and included in traffic assignment research with generalized travel time or cost with respect to paths. Compared with the researches mentioned above, through using the network dual equilibrium theorem to fit the generalized side constraints conveniently into one variational inequality model, we simplified the resolution and analysis of the model.

In the field of traffic assignment, how to determinate the actual used paths and relative path flows was not settled satisfactorily[8]. Based on the existing models' assumptions, researchers have proved there exists no unique solution of path flows. Even so, taking into consideration the actually

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used unique paths, researchers have designed some methods to determinate the path flows[9]. Because most of the methods mentioned above are based on maximum likelihood theory in probability or maximum entropy principle in thermodynamics, they can not interpret the real travel behaviors very well. Contrasting with the known methods, in this paper we took advantage of the data structure of the results of the specific model, and presented one method to solve the actually used paths and relative path flows that is easily applied in practice and with proper explanation.

2 Network Dual Equilibrium

Now consider a connected network with homogenesis flow. Corresponding to flow, there is the relative impedance, or so called cost. Flow and cost can be looked as a pair of dual variables. The basic elements of network include nodes and links. In the network, define $x=(x_1,\dots,x_i,\dots,x_n)^T$, $i=\{1,2,\dots,n\}$ as the flow vector and define $p=(p_1,\dots,p_j,\dots,p_n)^T$, $j=\{1,2,\dots,m\}$ as the cost vector. Here, the subscripts i and j denote sequence number of some elements in the network. The elements corresponding to flow and cost can be the same or different. Assume that there is a relationship $F(x,p)=0$ between flow and cost. Here $F(x,p)=0$ may be held as $p=p(x)$ or $x=x(p)$. In transportation network, the delay functions on links can be looked as such kind of relationship.

There are two basic ways to describe networks' equilibrium states. The first one is using variational inequalities to describe the equilibrium characteristic of networks as follows:

$$G(x^*,p^*)^T(x-x^*)+H(x^*,p^*)^T(p-p^*)\ge 0,\quad \{x,p\}\in\varphi. \tag{1}$$

Here ϕ is the constraints set of dual variables (x,p); $G(x^*,p^*)$ is the cost equilibrium expression corresponding to flow x when the network is in its equilibrium state $H(x^*,p^*)$ is the flow equilibrium expression corresponding to cost p in equilibrium state.

The other way is using projected dynamic systems theorem to describe the evolving history of network eqilibrium as follows:

$$\begin{cases}\dot{x}=\Pi_{\varphi_x}(x-B\cdot G(x,p));\\ \dot{p}=\Pi_{\varphi_p}(p-B\cdot H(x,p)).\end{cases} \tag{2}$$

Here $\dot{x}$ and $\dot{p}$ are the derivatives of flow and cost regarding to time, respectively; $\Pi_\phi(t)$ is the projection of variable t on set ϕ; B is a given positive definite matrix. ϕ_x denotes subset of ϕ after fixing the cost vector p. Both ways can be applied to a large extent in the researches on network equilibrium[8,10-11].

3 The model without side constraints based on network dual equilibrium

For every link of the transportation network there is a corresponding link flow. For every node of the network we can define a travel time. We define the total travel time from the origin of the given OD pair to the temporal node as the cost corresponding to this node.

At first, let us consider a network with only one OD pair. According to the dual equilibrium theorem, the flows at any node should satisfy

$$\Big(\sum_{j\in V_i} f_{ji}^*+g_i-\sum_{j\in W_i} f_{ij}^*\Big)\times(\pi_i-\pi_i^*)\ge 0,\quad \forall i\in I \tag{3}$$

In (3), I is the node set; g_i is the total flow generating from node i. If node i is an origin, g_i is positive; if i is a destination, g_i is negative; otherwise, g_i equals zero. f_{ij} denotes the flow on the directed link (i,j) with tail j and head i. π_i denotes the possible travel time from the origin of the unique OD pair to note i. The superscript "*" indicates that the corresponding variable takes its value of network equilibrium state. The set W_i denotes $\{j|(i,j)\in A\}$ and the set V_i denotes $\{j|(j,i)\in A\}$. Here A is the link set of the network. When the network stays at equilibrium, according to the definition of π_i, π_i^* is always greater than 0; if π_i^* is greater than 0, the flows at node i satisfy the flow conservation function. That is, the first item of the left side of inequality (3),

$$\sum_{j\in V_i} f_{ji}^*+g_i-\sum_{j\in W_i} f_{ij}^*,$$

should equal 0.

Let $c_{ij}(f_{ij})$ denote the travel time on link (i,j) with flow f_{ij}. The variational inequality of cost equilibrium on link (i,j) is

$$(\pi_i^*+c_{ij}(f_{ij}^*)-\pi_j^*)\times(f_{ij}-f_{ij}^*)\ge 0\quad \forall(i,j)\in A \tag{4}$$

According to inequality (4), if f_{ij}^* is greater than 0, $\pi_i^*+c_{ij}(f_{ij}^*)-\pi_j^*$ equals 0; but from the definitions of π_i and π_j, $\pi_i^*+c_{ij}(f_{ij}^*)-\pi_j^*$ is always greater than or equal to 0.

If there are more than one link or path connecting two specified nodes and there is travel flow between these two nodes, the inequality (4) shows that travelers will choose the path or link with the least travel time to traverse. This principle of path finding mentioned above is a generalized one from Wardrop's first principle, named as "local nearsighted" user equilibrium principle. Comparing with the old assumption-travelers need to know perfect information of the whole network, and then choose the shortest path at the whole network level. The "local nearsighted" user equilibrium principle conforms to the real situation very well[8].

Sum up inequality (3) on node set I and inequality (4) on link set A, and then add them, so we can obtain

$$\sum_i\sum_j(\pi_i^*+c_{ij}(f_{ij}^*)-\pi_j^*)\times(f_{ij}-f_{ij}^*)+$$
$$\sum_i\Big(\sum_{j\in V_i} f_{ji}^*+g_i-\sum_{j\in W_i} f_{ij}^*\Big)\times(\pi_i-\pi_i^*)\ge 0$$

Let $f\ge 0$ and $\pi\ge 0$ be flow vector and cost vector,