JOURNAL OF TRANSPORTATION
SYSTEMS ENGINEERING AND INFORMATION TECHNOLOGY
Volume 8, Issue 6, December 2008
Online English edition of the Chinese language journal

Cite this article as: J Transpn Sys Eng & IT, 2008, 8(6), 96−100.

RESEARCH PAPER

Emergency Evacuation Model and Algorithms

CHEN Yueming, XIAO Deyun*

Department of Automation, Tsinghua University, Beijing 100084, China

Abstract: A scientific and effective emergency evacuation plan plays an important role in improving the event reaction ability of the urban traffic system, as well as, saves rescue time and reduces property losses. Evacuation route construction and network distribution in each network junction are vital for evacuation planning problems. An optimal objective based on the shortest emergency time is established and the optimal solution is acquired using the Pontryagin minimum principle. The evacuation route construction algorithm and traffic flow assignment algorithm in each junction are employed to deliver the traffic flow in the evacuation area to a safe region rapidly and safely. The idea of feedback is introduced in the execution using real-time information to adjust and update the evacuation plan. The simulation result shows that the proposed model and algorithm can be effectively carried out in an emergency evacuation.

Key Words: emergency evacuation; dynamic traffic flow; Pontryagin minimum principle; state-feedback

1 Introduction

Whether they are predictable or not, man-made and natural disasters could both result in severe life loss and property damage. Emergency evacuation, which drives a mass movement of people and their properties from disaster-impacted areas to safe areas, has been studied and practiced as a major means of countermeasure to mitigate these calamitous consequences. Houston had first formulated a dissipation rate model for estimating evacuation time in which a statistical aggregate method was considered. Voorhess *et al*.[1–3] have proposed some other models and put them into practice, under different emergency conditions. It takes a considerable amount of computation to calculate the operating characteristics of individual vehicles. Little progress has been achieved from decades as a result of low computation power. Over the last decade, the rapid development of computer technology has greatly promoted the research of emergency evacuation and obtained many research results. These can be generally classified into mathematical, analytical[4–6], and simulation based[7–8] approaches. For a detailed discussion, the authors refer to reviews by Urbina and Wolshon[9]. The main contribution of this paper is that it proposes an optimal traffic assignment model based on the shortest emergency evacuation time, by determining traffic assignment adaptively, according to the time varying conditions, so that the evacuation strategy can respond to a disaster rapidly and effectively. Evacuation route construction algorithm and network distribution algorithm are employed to deliver the traffic flow rapidly and safely from the evacuation areas to safe regions.

This paper is organized as follows: the next section describes the emergency evacuation model, and the necessary conditions for optimal solution are derived using Pontryagin minimum principle. The evacuation route construction algorithm and traffic flow assignment algorithm in each junction are introduced in Section 3. Simulations are given in the next section, followed by concluding remarks and future study given in the last section of this paper.

2 Model formulation

The traffic network is represented by a directed graph $G(A, N)$, where A is the set of directed arcs and N is the set of nodes. Index a denotes an arc, index k represents an origin node or an intersection node, and index n is a destination node. $A(k)$ is a set of arcs whose tail node is k, and $B(k)$ is the set of arcs whose head node is k. The representation $g_k^n(t)$ is the rate of vehicles departing from node k to node n at time t, with t as a known nonnegative and continuous function of time. The representation $u_a^n(t)$ is the control variable denoting the entering rate of vehicles into arc a at time t and destined to

Received date: Mar 14, 2008; Revised date: June 8, 2008; Accepted date: July 9, 2008
*Corresponding author. E-mail: xiaody@mails.tsinghua.edu.cn

node n. $v_a^n(t)$ denotes the departing rate of vehicles from arc a at time t. $x_a(t)$ is the state variable denoting the number of queuing vehicles in arc a at time t, whereas, $x_a^n(t)$ is the destination-specific state variable.

$$x_a(t) = \sum_{n \in N} x_a^n(t), \quad \forall a \in A, \forall t \in [0, T] \tag{1}$$

A scientific and effective emergency evacuation traffic management plays an important role in improving and saving rescue time and reducing property loss in emergency conditions. This primarily depends on the efficient utilization of roadway capacities and intelligent transportation system (ITS) technologies, effective coordination of traffic management equipment and available emergency aid resources. This paper proposes an optimal traffic assignment model based on the shortest emergency evacuation time that responds to disasters rapidly and effectively and delivers the traffic rapidly and safely from disaster-impacted areas to safe areas, by determining traffic assignment adaptable to continuously changing road conditions. The constraint conditions are as follows:

(1) State of the queuing segment

The dynamic evolution of the state of the queuing segment of each arc for each destination is described by the first order nonlinear equation:

$$\int_0^t u_a^n(w)\mathrm{d}w = \int_0^t v_a^n(w)\mathrm{d}w + x_a^n(t) \tag{2}$$

and this is equivalent to

$$\frac{\mathrm{d}x_a^n(t)}{\mathrm{d}t} = u_a^n(t) - v_a^n(t), \forall a, n, t \tag{3}$$

Note that Eq. (3) describes the relationship between the state variable $x_a^n(t)$ and the control variables $u_a^n(t)$ and $v_a^n(t)$.

(2) Flow conservation constraint

Conservation of vehicles at each node for each destination is ensured by the mixed state-control equality constraint:

$$\sum_{a \in A(k)} u_a^n(t) = g_k^n(t) + \sum_{a \in B(k)} v_a^n(t), \forall k, n, t; k \neq n \tag{4}$$

$$\sum_{a \in A(n)} u_a^n(t) = 0, \forall n, t \tag{5}$$

(3) Flow constraint

The following equation describes the relationship between $x_a^n(t)$ and the arc exit-flow function $v_a^n(t)$:

$$x_a^n(t) = \int_t^{t+c_a(t)} v_a^n(w)\mathrm{d}w \tag{6}$$

where $c_a(t)$ is the travel time function. For convenience, Eq. (6) can be simplified to

$$v_a^n(t)(t + c_a(t) - t) = x_a^n(t)$$

then it can be described as:

$$v_a^n(t) = \frac{x_a^n(t)}{c_a^n(t)} \tag{7}$$

(4) Nonnegative constraint

$$x_a^n(t) \geq 0, u_a^n(t) \geq 0, \ \forall a, n, t \tag{8}$$

The dynamic system-optimal traffic assignment problem for emergency evacuation is then formulated as:

$$J = \min \sum_{a \in A} \int_0^T x_a(t)\mathrm{d}t \tag{9}$$

subject to

$$\begin{aligned}
&\frac{\mathrm{d}x_a^n(t)}{\mathrm{d}t} = u_a^n(t) - v_a^n(t), \ \forall n, t \\
&\sum_{a \in A(k)} u_a^n(t) = g_k^n(t) + \sum_{a \in B(k)} v_a^n(t), \ \forall k, n, t; k \neq n \\
&\sum_{a \in A(n)} u_a^n(t) = 0, \ \forall n, t \\
&x_a^n(t) \geq 0, u_a^n(t) \geq 0, \ \forall a, n, t
\end{aligned} \tag{10}$$

where

$$v_a^n(t) = \frac{x_a^n(t)}{c_a^n(t)}, \forall a, n, t \tag{11}$$

The conditions for optimal solution to the problem J will be derived. Pontryagin minimum principle is usually applied, which has been developed by Budelis and Bryson for the case where the state differential equations contain a time delay in the control variables. To this end, the augmented Hamiltonian function of problem J is then constructed as:

$$\begin{aligned}
H = &\sum_{a \in A} x_a(t) + \sum_{a \in A} \sum_{n \in N} \lambda_a^n(t) \left[u_a^n(t) - v_a^n(t) \right] \\
&+ \sum_{k \in N} \sum_{n \in N} \sigma_k^n(t) \left[\sum_{a \in B(k)} v_a^n(t) + g_k^n(t) - \sum_{a \in A(k)} u_a^n(t) \right]
\end{aligned} \tag{12}$$

where $\lambda_a^n(t)$ and $\sigma_k^n(t)$ are Lagrange multipliers. The necessary condition can be derived as follows:

$$\frac{\partial H}{\partial u_a^n(t)} = \lambda_a^n(t) - \sigma_k^n(t) \geq 0, \forall k, n, t, a \tag{13}$$

and

$$u_a^n(t) \frac{\partial H}{\partial u_a^n(t)} = u_a^n(t)(\lambda_a^n(t) - \sigma_k^n(t)) = 0, \forall k, n, t, a \tag{14}$$

$$\frac{\mathrm{d}\lambda_a^n(t)}{\mathrm{d}t} = -\frac{\partial H}{\partial x_a^n(t)} = -[1 - (\lambda_a^n(t) - \sigma_k^n(t)) \frac{1 - v_a^n(t) \dfrac{\mathrm{d}c_a(t)}{\mathrm{d}x_a(t)}}{c_a(t)}] \tag{15}$$

$$\forall k, n \ t; k \neq n, a \in B(k)$$

$$\frac{\partial x_a^n(t)}{\partial t} = \frac{\partial H}{\partial \lambda_a^n(t)} = u_a^n(t) - v_n^n(t) \geq 0, \forall n, t, a \tag{16}$$

$$\frac{\partial H}{\partial \sigma_k^n(t)} = \sum_{a \in B(k)} v_a^n(t) + g_k^n(t) - \sum_{a \in A(k)} u_a^n(t) = 0, \forall k, n, t; k \neq n \tag{17}$$

$$x_a^n(t) \geq 0, u_a^n(t) \geq 0, \forall a, n, t \tag{18}$$

$$x_a^n(0) = 0, \lambda_a^n(T) = 0, \forall a, n \tag{19}$$

$$u_a^n(t) = 0 \ \text{ if } \ \lambda_a^n(t) > \sigma_k^n(t), \ \forall k, n, t, a \tag{20}$$

$$u_a^n(t) \geq 0 \ \text{ if } \ \lambda_a^n(t) = \sigma_k^n(t), \ \forall k, n, t, a \tag{21}$$

$$\lambda_a^n(t) = u_k^n(t) \ \text{ if } \ u_a^n(t) > 0, \ \forall k, n, t, a \tag{22}$$

The control variable $u_a^n(t)$ is determined by the value of $\lambda_a^n(t) - \sigma_k^n(t)$. When $\lambda_a^n(t) = \sigma_k^n(t)$, the maximum principle of the model cannot determine the exact value of control variable $u_a^n(t)$, which falls into the singular case. When this situation happens, the singular solution can be obtained from the Legendre-Clebsch condition as also an attached condition