JOURNAL OF TRANSPORTATION
SYSTEMS ENGINEERING AND INFORMATION TECHNOLOGY
Volume 9, Issue 3, June 2009
Online English edition of the Chinese language journal

Cite this article as: J Transpn Sys Eng & IT, 2009, 9(3), 93−97.

RESEARCH PAPER

Elastic Bottleneck Equilibrium Model for Rail Transit Passengers at Rush Hours

HAO Jixiu[1,2], ZHOU Wei[1,*], PENG Hu[2], WANG Yuanqing[1]

1 Chang'an University, Xi'an 710064, China
2 China Academy of Transportation Sciences, Beijing 100029, China

Abstract: As the traditional bottleneck models are failed to be fully utilized in the rail transit equilibrium at rush hours, the paper proposes the elastic bottleneck concepts which can reflect the trip character in each station, and develops a bottleneck equilibrium traffic flow model. First, with extending the definition of congestion risk cost, a more reasonable travel cost model for different stations at different time is developed at rush hours. Then, the peak hours computing equation is formulated to investigate the characteristics of rush hours. Considering the elastic features of different stations, the elastic bottleneck equilibrium model at the rush hours is developed. Finally, after model solution, the similarity between the model result and land value of space diminishing is analyzed comparatively, which is used to demonstrate the rationality of the proposed elastic bottlenecks model.

Key Words: rush hours; rail transit; elastic bottleneck model; equilibrium model

1 Introduction

In the process of urban development in China, the industrial structure and consumption structure of the satellite town cannot be linked up with the residents' demand effectively, which causes very obvious tidal phenomenon of both commuting traffic on workdays and the entertainment traffic on holidays. Thus, how to make the reasonable fares and departure interval is closely related to whether the rail transit can play a full role and whether the operating costs can be reduced effectively.

The traffic bottleneck model is provided by Vickrey which is endogenous departure time choice model that every traveler has the same travel costs using deterministic queuing theory. It can be used to analyze the traveler choice behaviors of travel time and travel modes[1]. It was used more widely on path choice and traffic flow assignment overseas, and the researches on traffic modes choice were seldomly involved. In 2001, Kuwahara and Akamats extended the dynamic user-optimal assignment model and designed an algorithm to certificate the discrimination of different queues[2]. In 2003, Taylor provided a dynamic traffic assessment model based on time[3]. In 2005, Szeto and Lo analyzed the queue attribution and extension in the course of dynamic traffic assessment[4]. In 2006 and 2007, Mounce researched the dynamic traveler route choice model[5,6].

Domestic researchers focused on such three kinds of bottleneck models: 1. Aiming at the fixed through line between origin and destination, the model was established in which different moment travels have the same travel costs. The model provides evidence for the departure interval and price[7,8]. 2. Based on the first research achievement, it was established that public transport mixed with private cars could influence the private car flow. And different modes have same travel cost, hence, an equilibrium traffic flow model of different transport modes was establilshed[9]. 3. Based on the above two research results, the game relationship between the traffic managers and travelers was analyzed. Moreover, the two-level game model between the traffic managers and travelers and among travelers was also developed whose solution was also given[10].

The above-mentioned three research results analyzed the travel time distribution and game balance at rush hours, especially the public transit model mixed with the car simulates the interaction between public transit and the private car. However, the rail transit equilibrium model based on the

Received date: Jun 4, 2008; Revised date: Sep 11, 2008; Accepted date: Sep 15, 2008
*Corresponding author. E-mail: zhw59@hotmail.com

DOI: 10.1016/S1570-6672(08)60067-7

bottleneck could not be directly applied to the urban rail transit game equilibrium model, mainly because there exist great differences between its assumptions and practices: (1) In practice, the riderships of urban rail transit in each two stations are not ideally equal. The origin of the line is dispersed and the destination is gathering or the destination of the line is dispersed and the origin is gathering. For example: to the hub or station A near the urban centeral area, travelers from different stations gathered to station A through line L in A.M. Peaks, and dispersed from station A to other stations in P.M. peaks. (2) In an actual road section, bottleneck road section is the key of the bottleneck model, and the bottleneck of the rail transit is transmission capacity. From the theoretical analysis, assuming that the passengers of station B had made the train in full load conditions, then the passengers of station C could not get on the train at rush hours. It was not reasonable obviously. This phenomenon can be found in actual operation. For instance, some passengers have to wait for the next trip because of overcrowding in the stations of Tongzhoubeiyuan, Shuangqiao, and Communication University of Beijing Batong Lightrail in A.M. peak. Thus, the standards of "bottleneck" are different to each station. Because of this, the paper provides the elastic "bottleneck" model which is closer to reality, as well as simulates and analyzes the traffic flow conditions at rush hours.

2 Elastic bottleneck

While the passenger flow is at the equilibrium state, the travelers at the same station keep stable sensitivity to the rail transit bottleneck (transmission capacity). However, travelers at different stations have different sensitivities to the rail transit bottleneck caused by long time "reality simulation". In view of this, the paper provides the concept of the elastic bottleneck to expand the scope of application of the rail transit equilibrium model at rush hours.

The concept of elastic bottleneck is that in the same station, the number of passengers which get on the train is N_u, the number which get off is N_d, the number which cannot get on is N_w. If the same rail transit stops at different stations with $N_u – N_d \geq 0$, and $N_w > 0$, which means that the tidal traffic phenomenon will occur in this rail line at rush hours, and this train have the elastic bottleneck at different stations.

3 Model hypothesis and symbol definition

Hypothesis 1: there is a rail line L from point B to point A, in addition to terminal B and A, there is a point C on the line L.

Hypothesis 2: there are N travelers from B and C to A through line L at rush hours (N_B in station B; N_C in station C); and the given time is t^*.

Hypothesis 3: cost per travel time is α; penalty cost of early time is β; penalty cost of late time is γ.

Hypothesis 4: N_B passengers in station B depart in l_B groups; N_C passengers in station C depart in l_C groups.

Hypothesis 5: As the rail transit is affected slight by other transport modes, it can be consumed that the travel time from station B and station C to station A are T_B and T_C. Thus, the travel time is $T_{B–C}$ from station B to station C. t_0 and t_0' denote respectively the departure moment which the passengers of station B and station C arrived at station A on time (that is, the passengers arrived at time t^*).

Hypothesis 6: S_B is the rated passenger capacity of line L in station B. S_C is the rated passenger capacity of line L in station C (except the capacity in station B). H is the departure interval. p is the ticket price of line L.

Hypothesis 7: $R(t)$ is the congestion risk cost at time t. In station B, t_1 is the beginning of the crowded time; t_2 is the ending of the crowded time. In station C, t_1' is the beginning of the crowded time; t_2' is the ending of the crowded time. That is, station B is in the rush hours as $t \in [t_1, t_2]$; station C is in the rush hours as $t \in [t_1', t_2']$.

4 Calculation of travel cost

Travel costs of rail transit include running time costs, delay costs, waiting time costs, and the fares. In which the waiting time can be divided two parts: (1) waiting for the first train arrival; (2) waiting for the follow-up train to arrive caused by crowdedness. Thus, the travel costs of different travelers are not equal. The cost of waiting for the follow-up train can be splited to other traveler on average through the congestion risk costs[7].

Therefore, the travel costs by rail transit include running time costs T, delay costs D, congestion risk costs R, and tickets p. Then, the travel cost $C(t)$ of stations B and C can be calculated:

Station B:

$$C_B(t) = \alpha T_B + D_B(t) + R(t) + p \tag{1}$$

Station C:

$$C_C(t) = \alpha T_C + D_C(t) + R(t) + p \tag{2}$$

where, $D_B(t)$ is the delay costs of station B; and $D_C(t)$ is the delay costs of Station C.

Travelers in station B or station C all wish to arrive at station A at time t^*. In station B, early time traveler is $t^* – t – T_B$, late time traveler is $t + T_B – t^*$, the traveler on time $t_0 + T_B = t^*$. In station C, early time traveler is $t^* – t – T_C$, late time traveler is $t + T_C – t^*$, the traveler on time $t_0' + T_C = t^*$. As $T_B > T_C$, there are $t_1' = t_1 + T_{B-C}$; $t_2' = t_2 + T_{B-C}$; $t_0' = t_0 + T_{B-C}$. Then:

$$D_B(t) = \begin{cases} \beta(t^* - t - T_B), & t \in [t_1, t_0] \\ \gamma(T_B + t - t^*), & t \in (t_0, t_2] \end{cases} \tag{3}$$

$$D_C(t) = \begin{cases} \beta(t^* - t - T_C), & t \in [t_1', t_0'] \\ \gamma(T_C + t - t^*), & t \in (t_0', t_2'] \end{cases} \tag{4}$$