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On the analysis of Canadian Holstein dairy cow lactation curves using standard growth functions

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ABSTRACT

Six classical growth functions (monomolecular, Schumacher, Gompertz, logistic, Richards, and Morgan) were fitted to individual and average (by parity) cumulative milk production curves of Canadian Holstein dairy cows. The data analyzed consisted of approximately 91,000 daily milk yield records corresponding to 122 first, 99 second, and 92 third parity individual lactation curves. The functions were fitted using nonlinear regression procedures, and their performance was assessed using goodness-of-fit statistics (coefficient of determination, residual mean squares, Akaike information criterion, and the correlation and concordance coefficients between observed and adjusted milk yields at several days in milk). Overall, all the growth functions evaluated showed an acceptable fit to the cumulative milk production curves, with the Richards equation ranking first (smallest Akaike information criterion) followed by the Morgan equation. Differences among the functions in their goodness-of-fit were enlarged when fitted to average curves by parity, where the sigmoidal functions with a variable point of inflection (Richards and Morgan) outperformed the other 4 equations. All the functions provided satisfactory predictions of milk yield (calculated from the first derivative of the functions) at different lactation stages, from early to late lactation. The Richards and Morgan equations provided the most accurate estimates of peak yield and total milk production per 305-d lactation, whereas the least accurate estimates were obtained with the logistic equation. In conclusion, classical growth functions (especially sigmoidal functions with a variable point of inflection) proved to be feasible alternatives to fit cumulative milk production curves of dairy cows, resulting in suitable statistical performance and accurate estimates of lactation traits.
Key words: dairy cow, growth function, lactation curve, lactation trait

INTRODUCTION

A typical lactation curve increases rapidly from initial yield at calving to a peak then decreases gradually as lactation progresses. Several models with different functional forms have been proposed to represent daily milk yield versus time of lactation in dairy cows (e.g., Wood, 1967; Dijkstra et al., 1997; Pollott, 2000; López, 2008). Variations in the shape and form of the lactation curve stem from factors including genetic background, parity, diet, and other environmental influences (Wood, 1968, 1970, 1980). The models used to describe milk yield can be divided into 2 groups: mechanistic models, based on the biology of lactation aiming to describe the causative mechanisms underlying a specific shape (e.g., Dijkstra et al., 1997; Pollott, 2000), and empirical models, based on the mathematical representation of actual milk yield data and simply trying to describe the shape (e.g., Wood, 1967; Rook et al., 1993; Grossman et al., 1999). The choice of model is a balance between fitting properties and requirements for biological interpretation. For example, detailed mechanistic models (Pollott, 2000) have the advantage of parameters with biological interpretation, but can be difficult to fit and may give rise to parameter estimates showing large standard errors and multicollinearity. Conversely, more simple empirical models can rapidly converge to an acceptable solution and provide adequate fits to data, but lack a biological basis. A simple mechanistic model, such as the one proposed by Dijkstra et al. (1997), may provide such a balance between fitting properties and biological interpretation (Val-Arreola et al., 2004; Dematawewa et al., 2007).

Knowledge of the shape of the lactation curve is valuable in a management context, especially for decisions

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that are time-dependent. Modelers seek to find parametric descriptors of the shape of the lactation curve to predict characteristics including peak milk yield, time to peak, and persistency. For example, knowing when peak milk yield will occur can assist dairy farmers or managers in planning feeding strategies to maintain peak yield for as long as possible. Also, the persistency of milk yield is an important aspect of total yield. A cow with a flatter lactation curve compared with another cow with the same total 305-d milk production may experience less stress and have a higher feed efficiency (Grossman et al., 1999). Fathi-Nasri et al. (2008) reported the potential of the differentials of simple growth functions as equations for fitting monthly milk records of Holstein dairy cattle. Sigmoidal growth functions have been frequently applied to study somatic growth and population dynamics (Thornley and France, 2007; López, 2008). The shape of the cumulative milk production curve is similar to that of a growth curve; thus, various growth functions may be used to model the curve.

The cumulative milk production curve shows a smoother trajectory than that of the conventional lactation curve. It would be affected to a lesser extent if a few (sporadic) records are missing, and is less sensitive to the occurrence of outliers or abnormal recordings. Disturbances in the lactation curve can be highly influential when fitting classical lactation models, whereas the fit of cumulative milk production curves would be little affected by the occurrence of such occasional deviations. Furthermore, the lactation curve (daily milk yield vs. DIM) does not always follow the typical trajectory (standard curve), and other curve shapes have been observed. Nonstandard lactation curve shapes are not always easily fitted by conventional lactation models (Rekik and Gara, 2004; Macciotta et al., 2005; Silvestre et al., 2009). Regardless of the shape of the lactation curve, the cumulative milk production curve will always follow a monotonically increasing pattern that can be fitted using growth functions to represent the underlying trend of the lactation curve relating to the productive potential of the cow. The purpose of the current work was to investigate the suitability of 6 standard growth functions in describing the 305-d lactation curve of Holstein dairy cows from cumulative milk production data calculated from daily individual cow milking records.

MATERIALS AND METHODS

Data

The data analyzed consisted of approximately 91,000 daily milk yield records, corresponding to 122 first, 99 second, and 92 third parity individual lactation curves (for a total of 313 curves) of Holstein dairy cows collected at the Elora Dairy Research Centre (University of Guelph). Lactation length varied from 180 to 575 d, but longer lactations were truncated to 305 d for this analysis. Most of the curves (187/313) were up to 305 d, and only 20 curves (17 of them for third parity) corresponded to lactations of less than 250 d. In all cases milk yield was recorded daily. The database contained cow records of birth date, calving date, lactation number (parity), milking date, and daily milk yield. Cumulative milk production was calculated from daily milk yield records. The cows were fed 60:40 forage-to-concentrate as a TMR composed (per kilogram of DM) of 332 g of corn silage, 221 g of alfalfa silage, 55 g of hay, 202 g of high-moisture corn, and 189 g of custom supplement, formulated to meet NRC (2001) requirements for a lactating dairy cow. All cows were fed and milked twice daily (feeding at 0700 and 1300 h; milking at 0500 and 1500 h).

Model Fitting

Cumulative milk production generally follows a curvilinear trend that closely resembles the trend of a classical somatic growth curve (Figure 1). The 6 equations presented in Table 1, and usually fitted to growth curves, were used to describe the cumulative lactation curves. The first equation is the simple exponential function proposed by Mitscherlich (1909), often referred to as the Mitscherlich or monomolecular equation, and has no point of inflection, thus representing only a diminishing returns (nonsigmoidal) pattern. The Schumacher, Gompertz, and logistic equations are 3-parameter sigmoidal functions each with a fixed point of inflection occurring when cumulative production reaches a certain proportion of the upper asymptote; asymptotic cumulative milk production as DIM tend to infinity ($\boldsymbol{y_f}$), namely at $\frac{y_f}{e^2}$, $\frac{y_f}{e}$, and $\frac{y_f}{2}$, respectively. The Richards and Morgan equations have an extra parameter (shape constant n) and can be considered flexible functions which represent diminishing returns or sigmoidal profiles (the Richards equation is sigmoidal if $n > -1$, and the Morgan equation if $n > 1$) with a point of inflection at any y-value. The 4-parameter Richards equation encompasses the monomolecular, Gompertz, and logistic equations when n has a value of −1, 0, and 1, respectively. In the models described, $y(t)$ is cumulative milk production (kg), t is time of lactation or DIM, and y_0, y_f, t_0, t_h, k, D, n (all >0 except $n \geq -1$ in the case of Richards) are parameters that define the scale and shape of the curves, whereas $y'(t) = \mathrm{d}y/\mathrm{d}t$ is the daily