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A mathematical function to predict daily milk yield of dairy cows in relation to the interval between milkings

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ABSTRACT

The milk production of a dairy cow is characterized by lactation production, which is calculated from daily milk yields (DMY) during lactation. The DMY is calculated from one or more milkings a day collected at the farm. Various milking systems are in use today, resulting in one or many recorded milk yields a day, from which different calculations are used to determine DMY. The primary objective of this study was to develop a mathematical function that described milk production of a dairy cow in relation to the interval between 2 milkings. The function was partly based on the biology of the milk production process. This function, called the 3K-function, was able to predict milk production over an interval of 12 h, so DMY was twice this estimate. No external information is needed to incorporate this function in methods to predict DMY. Application of the function on data from different milking systems showed a good fit. This function could be a universal tool to predict DMY for a variety of milking systems, and it seems especially useful for data from robotic milking systems. Further study is needed to evaluate the function under a wide range of circumstances, and to see how it can be incorporated in existing milk recording systems. A secondary objective of using the 3K-function was to compare how much DMY based on different milking systems differed from that based on a twice-a-day milking. Differences were consistent with findings in the literature.
Key words: daily milk yield, milking interval, milking system, mathematical function

Technical Note

Knowing the daily milk production of dairy cows is necessary for animal management and selection in breeding programs. The measurement in use is daily milk yield (**DMY**), which is defined by the International Committee for Animal Recording (ICAR, 2011) as "milk production over 24 hours." Milking twice a day is a common practice in many countries. In that case, DMY is the sum of the yields over 2 milkings.

Currently, however, milkings are not always twice a day, but range from once a day (Mackenzie et al., 1990) to twice, 3 times, or more with robotic milking (Bouloc et al., 2003). Official milk-recording data, therefore, originate from herds with different milking systems and milk-recording systems and, therefore from different numbers of milk recordings per day. For each milking system, a method has been developed (ICAR, 2011) to predict DMY. These methods use correction factors derived from large sets of milk-recording data. Estimation of breeding values, moreover, is based on DMY. For the purpose of estimating breeding values, ideally the same prediction method for all milking systems should be used. For instance, when the same cow is milked 3 or more times a day instead of twice a day, the DMY will be higher, in most cases resulting in overestimation of the breeding value (Miglior, 2004).

The primary aim of this study was to investigate the relationship between the milk produced by a cow and the milking interval (the time between 2 milkings). Because DMY is traditionally based on 2 milkings a day, 2 milkings is the standard in recording systems (ICAR, 2011). An important parameter to calculate, therefore, is the milk production over an interval of 12 h. A mathematical function was developed to predict milk production over an interval of 12 h (720 min). A secondary aim was to compare how much DMY based on different milking systems differs from DMY based on a twice-a-day milking system. The new mathematical function was used for those comparisons.

Many mathematical models (Everett and Wadell, 1969; Lee and Wardrop, 1984; DeLorenzo and Wiggans, 1986; Hargrove, 1994; Cassandro et al., 1995; Liu et al., 2000; Klopčič, 2004) have been proposed to predict DMY from 1 or 2 milkings a day. These models did not use information based on the biological relationship

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between milk production and the interval in which the milk was produced. Neal and Thornley (1983) developed a dynamic simulation model for milk production in the mammary gland. The model was based on the number and activity of the secretory cells and the effects of hormones. Milk production, in relation to the interval between the current milking and the previous milking, showed an exponential increase at the beginning and later leveled off to an asymptote. This behavior was assumed to be the result of cell degradation and milk present in the udder. Brody (1945) showed milk yields and fat percentages for milking intervals between 1 and 36 h; data for milk and fat production were derived by visual inspection from Figure 21.3 of Brody (1945) and are presented in Table 1.

To develop a mathematical model to describe the relation between milk production (m_t) and preceding interval (t), 3 requirements were needed (Everett and Wadell, 1969; DeLorenzo and Wiggans, 1986; Liu et al., 2000): the function must be asymptotic ($m_t \rightarrow m_\infty$ as $t \rightarrow \infty$), it has to go through the origin ($m_0 = 0$ if $t = 0$), and it should have a minimum number of parameters.

Because milk production in the mammary gland is based on cell division, it must be considered a multiplicative process, and a logarithmic transformation should be used (Snedecor and Cochran, 1989): $\ln(m_t)$ or $\ln(m_t + 1)$, in which m_t is the milk production in kilograms over an interval of t minutes. The transformation of $\ln(m_t + 1)$ was chosen because it is 0 at $t = 0$ and the result is always a nonnegative value. The Michaelis-Menten function (Brown and Rothery, 1993) meets the requirements mentioned above and is suitable for this purpose, so that

$$\ln\left(m_t + 1\right) = \ln\left(m_\infty + 1\right)\left(\frac{t}{b + t}\right), \qquad [1]$$

where m_t is the milk production (kg) over an interval of t (min), parameter m_∞ is the asymptotic milk production of the cow (kg) for $t = \infty$, and parameter b (min) regulates the curvature. If $t = b$, then the value of $\ln(m_b + 1) = 0.5 \ln(m_\infty + 1)$.

Equation [1] was fitted to the milk data of Brody (1945) in Table 1, by using nonlinear regression (NLREG version 5.3; Sherrod, 2002). Results are presented in Figure 1. The graph shows an almost perfect fit ($R^2 = 0.99$). The estimate of m_∞ was 12.13 kg and that of b was 391 min.

To more easily interpret Equation [1], it was reorganized and reparameterized. Parameter m_{720}, which is the milk production over an interval of 720 min, replaced m_∞, and parameter k replaced parameter b, where k is the ratio $\ln(m_{720} + 1)/\ln(m_\infty + 1)$, a value between 0 and 1. If $k = 0$, the function is a straight line; if $k = 1$, the function is a constant. The result of reparameterization was

$$\ln\left(m_t + 1\right) = \ln\left(m_{720} + 1\right)\left(\frac{t}{720 + k\left(t - 720\right)}\right). \qquad [2]$$

Table 1. Milk and fat production for a cow with milking intervals of 1 through 36 h[1]

Interval		Milk		Fat	
h	min	lb.	kg	g	%
1	60	1.1	0.50	38	7.6
2	120	1.7	0.77	60	7.8
3	180	2.8	1.27	88	6.9
4	240	3.2	1.45	91	6.3
5	300	4.3	1.95	117	6.0
6	360	4.3	1.95	115	5.9
7	420	5.8	2.63	142	5.4
8	480	6.5	2.95	153	5.2
9	540	6.7	3.04	164	5.4
10	600	8.2	3.72	179	4.8
11	660	9.0	4.08	192	4.7
12	720	9.6	4.35	196	4.5
13	780	10.2	4.63	194	4.2
14	840	9.6	4.35	170	3.9
16	960	10.7	4.85	184	3.8
18	1,080	11.7	5.31	223	4.2
20	1,200	12.8	5.81	238	4.1
24	1,440	13.3	6.03	278	4.6
28	1,680	14.7	6.67	313	4.7
32	1,920	15.4	6.99	251	3.6
36	2,160	14.7	6.67	253	3.8

[1]Derived by visual inspection from Figure 21.3 of Brody (1945). Fat grams were calculated from fat percentage and milk yield.