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On sequences of Toeplitz matrices over finite fields

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In this paper we consider sequences $\left\{\mathcal{A}_{n}\right\}$ of Toeplitz matrices with entries in an arbitrary finite field $F$. The sequence $\left\{\mathcal{A}_{n}\right\}$ is uniquely determined by a pair of sequences $a_{0}, a_{1}, \ldots$ and $b_{1}, b_{2}, \ldots$ over $F$, so that $\mathcal{A}_{n}$ is the $(n+1) \times(n+1)$ matrix

$$
\left[\begin{array}{cccccc}
a_{0} & a_{1} & a_{2} & \ldots & a_{n-1} & a_{n}  \tag{1.1}\\
b_{1} & a_{0} & a_{1} & \ldots & a_{n-2} & a_{n-1} \\
b_{2} & b_{1} & a_{0} & \ldots & a_{n-3} & a_{n-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
b_{n} & b_{n-1} & b_{n-2} & \ldots & b_{1} & a_{0}
\end{array}\right]
$$

In particular, $\mathcal{A}_{n}$ lies in the top left corner (as well as the bottom right corner) of $\mathcal{A}_{n+1}$.

For each sequence $\left\{\mathcal{A}_{n}\right\}$ let $\nu=\left\{\nu_{n}\right\}$ be the corresponding sequence of nullities, i.e. $\nu_{n}$ is the nullity of $\mathcal{A}_{n}$. As in [1], where K. Culler and one of the authors considered nullity sequences of skew centro-symmetric matrices $\mathcal{A}_{n}$ (see also [10],[11]), here we ascertain the patterns of the nullity sequences which can occur for Toeplitz matrices. We present an elementary proof which shows that $\left\{\nu_{n}\right\}$ is a concatenation of strings of the form $0,0, \ldots, 0$ or $1,2, \ldots, d-1, d, d, \ldots, d, d-1, d-2, \ldots, 1,0$, where $d$ can repeat any finite number of times; or the nullity sequence may consist of a concatenation of finitely many strings of this type, followed by $1,2, \ldots$, see Theorem 1. These patterns were observed in [4] for sequences of Toeplitz matrices over $\mathbb{C}$. We analyze the structure of the (right) kernels of matrices which satisfy these patterns and use this to determine the number of matrices $\mathcal{A}_{0}, \ldots, \mathcal{A}_{n}$ which satisfy a specific nullity pattern, $\nu_{0}, \ldots, \nu_{n}$. We note that the structure of kernels of Toeplitz matrices over $\mathbb{C}$ is carried out in [4], and a similar analysis is carried

[^0]Abstract. For each non-negative integer $n$ let $\mathcal{A}_{n}$ be an $n+1$ by $n+1$ Toeplitz matrix over a finite field, $F$, and suppose for each $n$ that $\mathcal{A}_{n}$ is embedded in the upper left corner of $\mathcal{A}_{n+1}$. We study the structure of the sequence $\nu=\left\{\nu_{n}: n \in \mathbb{Z}^{+}\right\}$, where $\nu_{n}=\operatorname{null}\left(\mathcal{A}_{n}\right)$ is the nullity of $\mathcal{A}_{n}$. For each $n \in \mathbb{Z}^{+}$and each nullity pattern $\nu_{0}, \nu_{1}, \ldots, \nu_{n}$, we count the number of strings of Toeplitz matrices $\mathcal{A}_{0}, \mathcal{A}_{1}, \ldots, \mathcal{A}_{n}$ with this pattern. As an application we present an elementary proof of a result of D. E. Daykin on the number of $n \times n$ Toeplitz matrices over $G F(2)$ of any specified rank. 2000 MSC Classification 15A33, 15A57
Keywords: Toeplitz matrix, nullity sequence, rank, finite fields

## 1. Introduction

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