Accepted Manuscript

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PII:S0024-3795(18)30446-4DOI:https://doi.org/10.1016/j.laa.2018.09.013Reference:LAA 14728To appear in:Linear Algebra and its Applications

Received date:9 April 2018Accepted date:10 September 2018

Please cite this article in press as: G. Price, M. Wortham, On sequences of Toeplitz matrices over finite fields, *Linear Algebra Appl.* (2018), https://doi.org/10.1016/j.laa.2018.09.013

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ACCEPTED MANUSCRIPT

ON SEQUENCES OF TOEPLITZ MATRICES OVER FINITE FIELDS

GEOFFREY PRICE* AND MYLES WORTHAM

ABSTRACT. For each non-negative integer n let \mathcal{A}_n be an n+1 by n+1 Toeplitz matrix over a finite field, F, and suppose for each n that \mathcal{A}_n is embedded in the upper left corner of \mathcal{A}_{n+1} . We study the structure of the sequence $\nu = \{\nu_n : n \in \mathbb{Z}^+\}$, where $\nu_n = \text{null}(\mathcal{A}_n)$ is the nullity of \mathcal{A}_n . For each $n \in \mathbb{Z}^+$ and each nullity pattern $\nu_0, \nu_1, \ldots, \nu_n$, we count the number of strings of Toeplitz matrices $\mathcal{A}_0, \mathcal{A}_1, \ldots, \mathcal{A}_n$ with this pattern. As an application we present an elementary proof of a result of D. E. Daykin on the number of $n \times n$ Toeplitz matrices over GF(2) of any specified rank. 2000 MSC Classification 15A33, 15A57

Keywords: Toeplitz matrix, nullity sequence, rank, finite fields

1. INTRODUCTION

In this paper we consider sequences $\{\mathcal{A}_n\}$ of Toeplitz matrices with entries in an arbitrary finite field F. The sequence $\{\mathcal{A}_n\}$ is uniquely determined by a pair of sequences a_0, a_1, \ldots and b_1, b_2, \ldots over F, so that \mathcal{A}_n is the $(n+1) \times (n+1)$ matrix

| | a_0 | a_1 | a_2 | | a_{n-1} | a_n |
|-------|-------|-----------|-----------|----|-------------|-----------|
| | b_1 | a_0 | a_1 | | a_{n-2} | a_{n-1} |
| (1.1) | b_2 | b_1 | a_0 | | a_{n-3} | a_{n-2} |
| | : | : | | ·. | : | : |
| | b_n | b_{n-1} | b_{n-2} | | \dot{b}_1 | a_0 |

In particular, \mathcal{A}_n lies in the top left corner (as well as the bottom right corner) of \mathcal{A}_{n+1} .

For each sequence $\{A_n\}$ let $\nu = \{\nu_n\}$ be the corresponding sequence of nullities, i.e. ν_n is the nullity of \mathcal{A}_n . As in [1], where K. Culler and one of the authors con-8 sidered nullity sequences of skew centro-symmetric matrices \mathcal{A}_n (see also [10],[11]), 9 here we ascertain the patterns of the nullity sequences which can occur for Toeplitz 10 matrices. We present an elementary proof which shows that $\{\nu_n\}$ is a concatenation 11 of strings of the form $0, 0, \ldots, 0$ or $1, 2, \ldots, d - 1, d, d, \ldots, d, d - 1, d - 2, \ldots, 1, 0$, 12 where d can repeat any finite number of times; or the nullity sequence may consist 13 of a concatenation of finitely many strings of this type, followed by $1, 2, \ldots$, see 14 Theorem 1. These patterns were observed in [4] for sequences of Toeplitz matrices 15 over \mathbb{C} . We analyze the structure of the (right) kernels of matrices which satisfy 16 these patterns and use this to determine the number of matrices $\mathcal{A}_0, \ldots, \mathcal{A}_n$ which 17 satisfy a specific nullity pattern, ν_0, \ldots, ν_n . We note that the structure of kernels 18 of Toeplitz matrices over \mathbb{C} is carried out in [4], and a similar analysis is carried 19

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Date: 13 August 2018.

^{*} supported in part by the United States Naval Research Laboratory, Washington, DC.

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