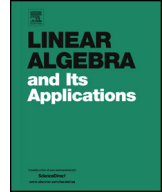




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Linear Algebra and its Applications

www.elsevier.com/locate/laa



On rank range of interval matrices



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ARTICLE INFO

Article history:

Received 3 January 2018
 Accepted 19 September 2018
 Available online 21 September 2018
 Submitted by R. Brualdi

MSC:
 15A99
 15A03
 65G40

Keywords:
 Interval matrices
 Rank

ABSTRACT

An interval matrix is a matrix whose entries are intervals in \mathbb{R} . Let $p, q \in \mathbb{N} \setminus \{0\}$ and let $\alpha = ([\underline{\alpha}_{i,j}, \bar{\alpha}_{i,j}])_{i,j}$ be a $p \times q$ interval matrix; given a $p \times q$ matrix A with entries in \mathbb{R} , we say that $A \in \alpha$ if $a_{i,j} \in [\underline{\alpha}_{i,j}, \bar{\alpha}_{i,j}]$ for any i, j . We establish a criterion to say if an interval matrix contains a matrix of rank 1. Moreover we determine the maximum rank of the matrices contained in a given interval matrix. Finally, for any interval matrix α with no more than 3 columns, we describe a way to find the range of the ranks of the matrices contained in α .

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1. Introduction

Let $p, q \in \mathbb{N} \setminus \{0\}$; a $p \times q$ interval matrix is a $p \times q$ matrix whose entries are intervals in \mathbb{R} ; let $\alpha = ([\underline{\alpha}_{i,j}, \bar{\alpha}_{i,j}])_{i,j}$ be a $p \times q$ interval matrix, where $\underline{\alpha}_{i,j}, \bar{\alpha}_{i,j}$ are real numbers with $\underline{\alpha}_{i,j} \leq \bar{\alpha}_{i,j}$ for any i and j ; given a $p \times q$ matrix A with entries in \mathbb{R} , we say that $A \in \alpha$ if $a_{i,j} \in [\underline{\alpha}_{i,j}, \bar{\alpha}_{i,j}]$ for any i, j . In this paper we investigate about the range of the ranks of the matrices contained in α .

There are several papers studying when an interval $p \times q$ matrix α has full rank, that is when all the matrices contained in α have rank equal to $\min\{p, q\}$. For any $p \times q$ interval

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matrix $\alpha = ([\underline{\alpha}_{i,j}, \overline{\alpha}_{i,j}])_{i,j}$ with $\underline{\alpha}_{i,j} \leq \overline{\alpha}_{i,j}$, let $\text{mid}(\alpha)$, $\text{rad}(\alpha)$ and $|\alpha|$ be respectively the midpoint, the radius and the modulus of α , that is the $p \times q$ matrices such that

$$\text{mid}(\alpha)_{i,j} = \frac{\underline{\alpha}_{i,j} + \overline{\alpha}_{i,j}}{2}, \quad \text{rad}(\alpha)_{i,j} = \frac{\overline{\alpha}_{i,j} - \underline{\alpha}_{i,j}}{2}, \quad |\alpha|_{i,j} = \max\{|\underline{\alpha}_{i,j}|, |\overline{\alpha}_{i,j}|\}$$

for any i, j . The following theorem characterizes full-rank square interval matrices:

Theorem 1 (Rohn, [7]). *Let $\alpha = ([\underline{\alpha}_{i,j}, \overline{\alpha}_{i,j}])_{i,j}$ be a $p \times p$ interval matrix, where $\underline{\alpha}_{i,j} \leq \overline{\alpha}_{i,j}$ for any i, j . Let $Y_p = \{-1, 1\}^p$ and, for any $x \in Y_p$, denote by T_x the diagonal matrix whose diagonal is x .*

Then α is a full-rank interval matrix if and only if, for each $x, y \in Y_p$,

$$\det(\text{mid}(\alpha)) \det(\text{mid}(\alpha) - T_x \text{rad}(\alpha) T_y) > 0.$$

See [7] and [8] for other characterizations. The following theorem characterizes full-rank $p \times q$ interval matrices, see [9], [10], [12]:

Theorem 2 (Rohn). *A $p \times q$ interval matrix α with $p \geq q$ has full rank if and only if the system of inequalities*

$$|\text{mid}(\alpha) x| \leq \text{rad}(\alpha) |x|, \quad x \in \mathbb{R}^q$$

has only the trivial solution $x = 0$.

A problem which can be connected with the quoted ones is the one of the partial matrices: let K be a field; a partial matrix over K is a matrix where only some of the entries are given and they are elements of K ; a completion of a partial matrix is a specification of the unspecified entries. We say that a submatrix of a partial matrix is specified if all its entries are given. The problem of determining whether, given a partial matrix, a completion with some prescribed property exists and related problems have been widely studied: we quote, for instance, the papers [1], [2], [14]. In particular there is a wide literature about the problem of determining the maximal and the minimal rank of the completions of a partial matrix.

In [2], Cohen, Johnson, Rodman and Woerdeman determined the maximal rank of the completions of a partial matrix in terms of the ranks and the sizes of its maximal specified submatrices; see also [1] for the proof. The problem of determining the minimal rank of the completions of a partial matrix seems more difficult and it has been solved only in some particular cases, see for instance the paper [13] for the case of triangular matrices. We quote also the paper [3], where the authors establish a criterion to say if a partial matrix has a completion of rank 1.

Also for interval matrices, the problem of determining the minimal rank of the matrices contained in a given interval matrix seems much more difficult than the problem of

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