Spectral properties of general hypergraphs

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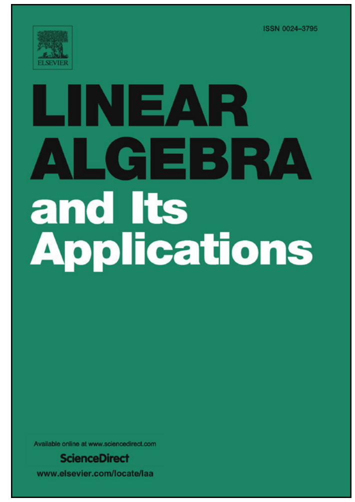

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Abstract

In this paper, we investigate spectral properties of the adjacency tensor, Laplacian tensor and signless Laplacian tensor of general hypergraphs (including uniform and non-uniform hypergraphs). We give some properties for the spectral radius of hypergraphs, and obtain spectral upper bounds for the chromatic number of hypergraphs. The odd-bipartiteness of hypergraphs can be recognized from the spectrum. We give a relation between the analytic connectivity and edge connectivity of a hypergraph, and show that a hypergraph with even rank is connected if and only if its analytic connectivity is larger than 0. We also give some relations between the analytic bipartiteness and odd-bipartiteness of hypergraphs.

Keywords: Hypergraph, Adjacency tensor, Laplacian tensor, Spectrum, Analytic connectivity, Analytic bipartiteness
AMS classification: 05C65, 05C50, 15A69, 15A18

1. Introduction

Let $V(H)$ and $E(H)$ denote the vertex set and the edge set of a hypergraph H, respectively. A hypergraph G satisfying $V(G) \subseteq V(H), E(G) \subseteq E(H)$ is called a *sub-hypergraph* of H. The *degree* of a vertex i of H is defined as $d_i = |E_i|$, where E_i denotes the set of edges containing i. The *rank* and *co-rank* of H is the maximum and minimum cardinality of an edge in H, respectively [1]. Let $r(H)$ and $cr(H)$ denote the rank and co-rank of H, respectively. If $r(H) = cr(H) = k$, then H is called *k-uniform*. 2-uniform hypergraphs are ordinary graphs. A *path* in a hypergraph H is defined to be an alternating sequence $u_1 e_1 u_2 \cdots u_l e_l u_{l+1}$, where $u_1, \ldots, u_{l+1}$

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