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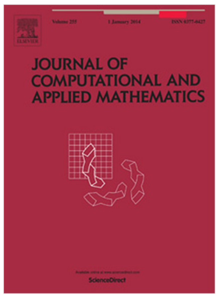

On the convergence of bivariate order statistics: Almost sure convergence and convergence rate

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On the convergence of bivariate order statistics: almost sure convergence and convergence rate

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Abstract

We study the convergence of bivariate order statistics, and get the almost sure convergence and convergence rate, generalizing one of main results in Huang et al.(2013). To be more precise, we prove the almost sure convergence of bivariate order statistics without the positive quadrant dependent (**PQD**) condition mentioned in Huang et al.(2013), and extend this result to a more general case. We also give the bound on the Kolmogorov distance between the distributions of bivariate order statistics and its limit. Our results provide an approximate algorithm for sampling from complicated structures, verified by some examples in the last section of this paper.

Keywords: Bivariate order statistics, Fréchet-Hoeffding bounds, Almost sure convergence, Kolmogorov distance, Sampling theory
2000 MSC: 62H10, 62G30

1. Introduction

Constructing a joint distribution with specific marginals and correlation has been a challenging problem since 1930s. This issue is widely used in numbers of fields including physics [1], economics [2], risk analysis [3], financial engineering [4], reliability theory [5], actuary science [6] etc.

To solve this problem, a well known and pioneering method called the FGM distribution was proposed, dating back to Eyraud [7], Farlie [8], Gumbel [9] and Morgenstern [10].This method is very simple but not so feasible, mentioned by Schucany et al [11]. It was extended to Sarmanov and Lee's distributions [12] later. Very recently Baker [13] introduced a new method to cope with this issue based on order statistics, which was extended by Bayramoglu's distributions [14] with an alternative approach later.

Let $(X_1, Y_1), (X_2, Y_2)..., (X_n, Y_n)$ be n independent copies of (X, Y) obeying a (possible dependent or independent) bivariate distribution $H(x, y)$ with marginal distributions $F(x) = P(X_1 \le x)$ and $G(y) = P(Y_1 \le y)$ respectively. Let $X_{1,n} \le X_{2,n} \le \cdots \le X_{n,n}$ and $Y_{1,n} \le Y_{2,n} \le \cdots \le Y_{n,n}$ be the order statistics of $\{X_i\}_{i=1}^n$ and $\{Y_j\}_{j=1}^n$, respectively. Write $X_{k,n} \sim F_{k,n}$ for the kth smallest order statistics of the random sample $\{X_i\}_{i=1}^n$ from F. Likewise, let $Y_{k,n}$ and $G_{k,n}$ for sequence $Y_i \sim G$.

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