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MODEL ORDER REDUCTION FOR PARAMETRIZED NONLINEAR HYPERBOLIC PROBLEMS AS AN APPLICATION TO UNCERTAINTY QUANTIFICATION

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Abstract. In this work, we present a model order reduction (MOR) technique for hyperbolic conservation laws with applications in uncertainty quantification (UQ). The problem consists of a parametrized time dependent hyperbolic system of equations, where the parameters affect the initial conditions and the fluxes in a non–linear way. The procedure utilized to reduce the order is a combination of a Greedy algorithm in the parameter space, a proper orthogonal decomposition (POD) in time and empirical interpolation method (EIM) to deal with non–linearities [18]. We provide under some hypothesis an error bound for the reduced solution with respect to the high order one. The algorithm shows small errors and savings of the computational time up to 90% in the UQ simulations, which are performed to validate the algorithm.

Key words. Reduced order modeling, reduced basis, nonlinear hyperbolic problems, UQ, empirical interpolation method, POD-Greedy, PODEI, residual distribution.

AMS subject classifications. 65M08, 65M15, 65J15, 76L05, 35L65, 35R60

1. Introduction. Parametrized partial differential equations (PPDE) have received in the last decades an increasing amount of attention from research fields as engineering and applied sciences. All these domains have in common the dependency of the PPDE on the input parameters, which are used to describe possible variations in the solution, initial conditions, source terms and boundary conditions, to name just a few. Hence, the solutions of these problems are depending on a large number of different input values, as in optimization, control, design, uncertainty quantification, real time query and other applications. In all these cases, the aim is to be able to evaluate in an accurate and efficient way an output of interest when the input parameters are varying. This will be very time consuming or can even become prohibitive when using high-fidelity approximation techniques, such as finite element (FE), finite volume (FV) or spectral methods. For this kind of problems, model order reduction (MOR) techniques are used, in order to replace the high-fidelity problem by one featuring a much lower numerical complexity. A key ingredient of MOR are the reduced basis (RB) methods, which allow to produce fast reduced surrogates of the original problem by only combining a few high-fidelity solutions (*snapshots*) computed for a small set of parameter values [27, 39, 23]. The most common and efficient strategies available to build a reduced basis space are the proper orthogonal decomposition (POD) and the greedy algorithm. These two sampling techniques have the same objective but in very different approach forms: the POD method is most often applied only in one dimensional (1D) space and mostly in conjunction with (Petrov-)Galerkin projection methods, in order to build reduced-order models (ROM) of time-dependent problems [31, 43], but also in the context of parametrized systems [10, 11, 29, 50]. The disadvantage of this method is that it relies on the singular value decomposition (SVD) of a large number of snapshots, which might entail a severe computational cost. On the other side, greedy algorithm [41, 42, 46] represents an efficient alternative to POD and is directly applicable in the multi-dimensional parameter domain. The algorithm is based on an iterative sampling from the parameter space fulfilling at each step a suitable optimality criterion that relies on a posteriori error estimates.

A first challenge in the context of ROM deal with unsteady problems, so implicitly the exploration of a parameter-time framework is needed. In this case, the sampling strategy to construct reduced basis spaces for the time-dependent problem is POD-greedy [24] and is based on combin-