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On fast multipole methods for Volterra integral equations with highly oscillatory kernels[1]

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Abstract. This paper explores the fast multipole methods (FMMs) to accelerate the approximation for weakly singular Volterra integral equations with highly oscillatory trigonometric kernels. By constructing the fast translation path, the FMM is utilized to speed up the iterative method, which reduces the complexity from $O(N^2)$ to $O(N)$. Especially, we use the collocation method to discretize the Volterra integral equation with constants and linear elements respectively, then apply the GMRES to solve the dense and non-symmetric linear system. In addition, the highly oscillatory integrals derived from the algorithm are calculated effectively by the steepest descent method. The proposed method shows that the numerical solutions become more accurate as the frequency increases. Both of the optimal convergence rate on truncation and the error bounds analysis are represented in the end.

Keywords. Fast multipole method. Volterra integral equation. Weak singularity. Highly oscillatory integrals. Steepest descent method

1 Introduction

Integral equations play an important role in solving many mathematical problems in physics and engineering [1, 2, 11]. For example, the Helmholtz equation

$$\triangle u + \omega^2 u = 0 \quad \text{in} \quad R^d\backslash\bar{\Omega}, \quad d = 2,3 \tag{1.1}$$

is a classical model to describe the scattering of time-harmonic acoustic or electromagnetic waves problems. Subjected to appropriate boundary conditions, it can be reformulated as an integral equation

$$u(x) = \int_\Gamma \left\{ u(y)\frac{\partial G(x,y)}{\partial n(y)} - G(x,y)\frac{\partial u}{\partial n(y)} \right\} ds(y), \quad x \in R^d\backslash\bar{\Omega} \tag{1.2}$$

with

$$G(x,y) = \begin{cases} \frac{i}{4}H_0^{(1)}(\omega|x-y|) & d=2 \\ \frac{1}{4\pi}\frac{e^{i\omega|x-y|}}{|x-y|} & d=3, \end{cases}$$

where the kernel is highly oscillatory when $\omega \gg 1$ and (weakly) singular [3, 4, 5, 6]. Meanwhile, the second kind Volterra integral equations

$$y(x) + \int_0^x \frac{K(x,t,\omega)}{(x-t)^\alpha} y(t)dt = f(x), \quad x \in [0,1], \quad 0 < \alpha < 1, \tag{1.3}$$

offers a wide range of applications in mathematical physics [7, 8, 9, 10, 11]. Here $K(x,t,\omega)$ is a continuous function and $f(x)$ is a given function. Particularly, when $K(x,x,\omega) \neq 0$, (1.3) is weakly singular. In 2007 and 2010, Brunner et al. gave two comprehensive surveys and proposed some open problems

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