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Several new performance measures for Markov system with stochastic supply patterns and stochastic demand patterns

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ABSTRACT

A system consisting of supply and demand is considered in this paper. Both the supply patterns and demand patterns are random. Thus, both the supply and the demand are modeled by Markov processes. Both the state space of the supply and the demand are not binary (on/off), and they are partitioned into several 'levels' of functionality. The performance measure considered here is the probability that the demand is met by the corresponding 'level' supply. A closed form expression for the performance measure is obtained by using aggregated stochastic process theory and Kronecker matrix operations. In the meanwhile, the probability density function of a cycle time and the customer's demand can be met in this cycle has also been given. Finally, a numerical example is given to illustrate the results obtained in this paper.

1. Introduction

Markov repairable systems have received considerable attention in the literature during the past decades, since the sojourn times of the system in every state are exponentially distributed and the calculation are tractable. A lot of Markov repairable systems were modeled and their reliability indexes were concluded in Barlow and Proschan [1], and others. In terms of Ion-Channel modeling theory (see Colquhoun and Hawkes [2] for details), Cui et al. [3] modeled Markov repairable systems with history-dependent up and down states, in which some states are changeable in the sense that whether those physical states are up and down depends on the immediately preceding state of the system evolution process. Wang and Cui [4] extended the model into the semi-Markov repairable system with history-dependent up and down states which is more realistic than the previous one. Wang et al. [5] studied a Markov repairable system with stochastic regimes switching and got the distribution of up time of the system. Time interval omission problem is very important in the fields of Ion-Channel theory and reliability. Zheng et al. [6] built a new model which is based on a single-unit Markov repairable system first. In that model, if a short repair time does not affect the system operating or throughput, the system could be thought of as operating during this repair time. Bao and Cui [7] further extended the work of Zheng et al. [6] to series Markov repairable system with neglected or delayed failures. In fact, the time interval omission is considered from the customer side, because the shorter repair time interval may not affect the customer using. Cui et al. [8] and Liu et al. [9] further discussed the time interval omission problem by using aggregated stochastic process method. The environment condition in which the system operates may be influence the performance of the system. Hawks et al. [10] studied the reliability of the system operating in alternative environments. Based on the concept of sparse connection, Zhao et al. [18] introduced three new start-up demonstration tests which are more efficient and effective than previous models.

In real life, the supply is almost random. Csenki [11,12] investigated a Markov system that is consisted of the supply and demand, in which the supply patterns are random and the demand patterns is deterministic. The system performance measure which was discussed is the probability that the supply will meet demand throughout the union of a given disjoint set of k time intervals. Cui et al. [13] considered a Markov system consisting of random supply and deterministic demand, and the multi-point, multi-interval and mixed multi-point-interval availabilities were obtained by using aggregated stochastic process theory. In terms of the model of Cui et al. [13], Du et al. [14] investigated the joint availability for k-out-of-n and consecutive k-out-of-n points and intervals.

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Most of the former researchers always supposed that the demand is deterministic and the supply is random, but little work has considered the random demand patterns in reliability field. In real life, the demand is not always deterministic, for example, person demand for the ambulances and the fire tracks are often random. Lisnianski [15] considered a multi-state system (MSS) in which the demand is a random process. A generalized reliability measure for MSS was discussed in the paper. Csenki [16] and Liu [17] also considered the stochastic demand patterns. In Csenki [16], the contribution is

(i) to model the stochastic supply processes and the stochastic demand processes,
(ii) to get the performance measure $C(k)$, i.e., the probability that the demand is met by supply in the first k random demand time intervals,
(iii) to put forward some avenues for future work.

In Liu [17], the supply and demand processes are identical to that in Csenki [16], but the system performance measure which is considered in this paper is different from that in Csenki [16]. The performance measure considered in this paper is the probability that the demand is met by the supply during a given time interval $[0, t]$, a general interval $[a, b]$ and multiple intervals $[a_1, b_1], [a_2, b_2], \ldots, [a_m, b_m]$.

In the present paper, both the supply patterns and demand patterns are random, and the state space of the supply and demand are not binary (on/off), and they are partitioned into several 'levels' of functionality. The performance measure considered here is the probability that the demand is met by the corresponding 'level' supply. In the meanwhile, the probability density function of a cycle time and the customer's demand can be met in this cycle has also been given in the present paper. In fact, the performance measure had been putted forward in future work in Csenki [16]. However, the problem has not been solved by researchers until now. A closed form expression is obtained for the system performance measure in this paper.

The paper is structured as follows. The coming section introduces model assumptions and some basic knowledge used in this paper. In Section 3, the system performance measure is obtained by using aggregated stochastic process theory and Kronecker matrix operations. In Section 4, a numerical example is given to illustrate the results obtained in this paper. Conclusion remarks are offered in the last section.

2. Model and related basic knowledge

2.1. The supply process

We suppose that the supply is modeled by an irreducible homogenous continuous time Markov chain $\{X(t), t \geq 0\}$,with a transition rate matrix $\mathbf{\Lambda}$ and a finite state space $\mathbf{S} = \{1, 2, \ldots, n_1, n_1+1, \ldots, n\}$ which can be partitioned into two disjoint sets: supply (or work) states $\mathbf{U} = \{1, 2, \ldots, n_1\}$ and no supply (or failure) states $\mathbf{D} = \{n_1+1, \ldots, n\}$, i.e., $\mathbf{S} = \mathbf{U} \cup \mathbf{D}$, $\mathbf{U} \cap \mathbf{D} = \phi$. Let $\mathbf{S}_l \subseteq \mathbf{U}$, $l = 1, 2, \ldots, k$, and $\mathbf{S}_l$, $l = 1, 2, \ldots, k$ need not be disjoint. When the supply process $\{X(t), t \geq 0\}$ sojourns in different state sets, it indicates that the system has different output powers or performance levels. The initial probability vector is α.

2.2. The demand process

We suppose that the demand is modeled by an irreducible homogenous continuous time Markov chain $\{Y(t), t \geq 0\}$, with a transition rate matrix $\mathbf{\Psi}$ and a finite state space $\mathbf{T} = \{a_1, a_2, \ldots, a_k, a_{k+1}, \ldots, a_m\}$ which can be partitioned into two disjoint sets: the set of neutral states $\mathbf{N} = \{a_1, a_2, \ldots, a_k\}$ and its complement $\mathbf{A} = \{a_{k+1}, \ldots, a_m\}$, the set of active states, i.e., $\mathbf{T} = \mathbf{N} \cup \mathbf{A}$, $\mathbf{N} \cap \mathbf{A} = \phi$. Let $\mathbf{T}_r \subseteq \mathbf{A}$, $r = 1, 2, \ldots, k$, and $\mathbf{T}_r$, $r = 1, 2, \ldots, k$ need not be disjoint. When the demand process $\{Y(t), t \geq 0\}$ is in different state sets, it indicates that customers have different needs, When the demand process $\{Y(t), t \geq 0\}$ sojourns in $\mathbf{N}$, there is no expectation concerning the facility from the customers side, the supply process $\{X(t), t \geq 0\}$ can be anywhere in $\mathbf{S}$; When the demand process $\{Y(t), t \geq 0\}$ sojourns in set $\mathbf{T}_r$, the supply process $\{X(t), t \geq 0\}$ is hoped by the customers to be in the set of active states $\mathbf{S}_r$. We assume that the demand process is independent of the supply process. The initial probability vector is β.

2.3. The combined process of the supply process and the demand process

Let $Z(t) = (X(t), Y(t))$, then the combined, bivariate stochastic process $\{Z(t), t \geq 0\}$ models the interaction between the supply and demand. Its state space is $\mathbf{S} \times \mathbf{T}$, the Cartesian product of the individual state spaces $\mathbf{S}$ and $\mathbf{T}$. Because of the independence of the component processes, $\{Z(t), t \geq 0\}$ is a Markov process with transition rate matrix

$$\mathbf{\Gamma} = \mathbf{\Lambda} \oplus \mathbf{\Psi} = \mathbf{\Lambda} \otimes \mathbf{I}_{TT} + \mathbf{I}_{SS} \otimes \mathbf{\Psi}, \tag{1}$$

where $\oplus$ and $\otimes$ denote the matrix operations Kronecker sum and Kronecker product, respectively. $\mathbf{\Gamma}$ is a square matrix of size $(|\mathbf{S}| \times |\mathbf{T}|) \times (|\mathbf{S}| \times |\mathbf{T}|)$.

The initial probability vector of $\{Z(t), t \geq 0\}$ is $\boldsymbol{\alpha} \otimes \boldsymbol{\beta}$, it is a row vector of length $|\mathbf{S}| \times |\mathbf{T}|$.

Let

$$P_{\varepsilon\varepsilon}(t) = \left(P\left\{Z(t) = j, Z(u) \in \varepsilon, u \leq t | Z(0) = i\right\}\right), \quad i, j \in \varepsilon,$$

and by Colquhoun and Hawkes [2], we have

$$\mathbf{P}_{\varepsilon\varepsilon}(t) = \exp(\mathbf{\Gamma}_{\varepsilon\varepsilon} t), \tag{2}$$

where $\mathbf{P}_{\varepsilon\varepsilon}(t)$ denotes the probability that the repairable system remains within the states $\boldsymbol{\varepsilon}$ throughout the time from 0 to t.

$$\mathbf{G}_{\varepsilon\omega}(t) = \mathbf{P}_{\varepsilon\varepsilon}(t)\mathbf{\Gamma}_{\varepsilon\omega}. \tag{3}$$

Elements in matrix $\mathbf{G}_{\varepsilon\omega}(t)$ are $g_{ij}(t), i \in \boldsymbol{\varepsilon}, j \in \boldsymbol{\omega}$, which are defined as follows,

$$g_{ij}(t) = \lim_{\Delta t \to 0} P\{\text{stay in}\varepsilon \text{ from time 0 to } t, \text{and leave} \boldsymbol{\varepsilon} \text{ for state} j \text{ between time } t \text{ and} t + \Delta t | \text{ in state} i \text{ at time } 0\}/\Delta t.$$

The Laplace transforms of $\mathbf{P}_{\varepsilon\varepsilon}(t)$ and $\mathbf{G}_{\varepsilon\omega}(t)$ are as follows, respectively,

$\mathbf{P}^*_{\varepsilon\varepsilon}(s) = (s\mathbf{I} - \mathbf{\Gamma}_{\varepsilon\varepsilon})^{-1}$, and $\mathbf{G}^*_{\varepsilon\omega}(s) = (s\mathbf{I} - \mathbf{\Gamma}_{\varepsilon\varepsilon})^{-1}\mathbf{\Gamma}_{\varepsilon\omega}$.

3. System performance measures

Let $0 = t_0 < t_1 < t_2 < \cdots < t_k$, then we can get k time intervals, i.e., $I_l = (t_{l-1}, t_l]$, $l = 1, 2, \ldots, k$. Furthermore, we require that the supply process $\{X(t), t \geq 0\}$ resides in state set $\mathbf{S}_l$ during time interval I_l. The probability of the event happening shall be denoted by

$$A(t_1, t_2, \ldots, t_k; \mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_k) = P\left(\bigcap_{l=1}^{k} \{X(t) \in \mathbf{S}_l, \text{for} \forall t \in (t_{l-1}, t_l]\}\right). \tag{4}$$

The formula shows that customers have different demand expectations for systems in different time intervals. Now we are interested in the probability that the customers demands are met from time 0 to t_k.