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Stochastic evaluation methods of a multi-state system via a modular decomposition

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ABSTRACT

Theoretical examinations about multi-state systems started around the publication of Barlow and Proschan [1], which has totally summarised the achievements about binary state systems by many researchers. Recently, a model of a multi-state system is tried to be applied for solving real problems and some authors have published books written from a practical point of view. See Natvig [16], Lisnianski and Levitin [9] and Lisnianski et al. [14], in which we may find real application of the results. In the reliability theory, evaluation of reliability performances of real systems and developing effective evaluation methods are important theme of reliability researchers. In this paper, we propose some evaluation methods of multi-state systems, using basic elements corresponding to the minimal path and cut state vectors of the binary state systems. We especially show convenient stochastic bounds via a modular decomposition. State spaces are assumed to be partially ordered sets, which means that the model proposed in this paper is one of the extremely extended models from the binary case.

1. Introduction

One of the most important issues in reliability theory is to explain order theoretic and probabilistic relations between a system and components, by which efficient reliability evaluation methods have been proposed. As for various large scale systems usually shown in the modern society, the reliability evaluation is faced with computational burden along with the increasing number of components concocting the system and complexity of logical connection among them. In these circumstance, practical and simple computational methods are desired for the reliability evaluation. It is one of the answers for the above problem to give stochastic upper and lower bounds for the reliability of systems. In this paper, we present some stochastic bounds for multi-state systems having partially ordered state spaces.

Many works have been performed for binary state systems, and the fruit of these works are applied to solve practical reliability problems. For example see Mine [15], Birnbaum and Esary [3], Birnbaum et al. [4], Esary and Proschan [5]. These works have been totally summarised by Barlow and Proshcan [1].

Systems and their components, however, could practically take many intermediate performance levels between perfectly functioning and complete failure states, and furthermore several states sometime can not be compared with each other. For a simple example, when a component of a production machine is characterised by two physical parameters, temperature and pressure, practically well observed situation, and we here assume that the optimal ones are 250 °C and 10*MPa*. The state of the component is denoted by a pair of these two characteristics (T, P). For two states (150, 12) and (260, 7), of course (150, 12) < (250, 10) and (260, 7) < (250, 10) hold, where (150, 12) < (250, 10), for example, means that the state (250, 10) is better than (150, 12). But these two states could not be ordered with each other, in other words, we may not state which state (150, 12) or (260, 7) is better or worse than the other, if these two sates relate to different inferior produced goods. Then this order < is not the usual order defined between tow real tow-dimensional vectors. Such a situation can be well formulated by using the general order theoretical concept. Hence a multi-state reliability model with partially ordered state spaces is required for understanding and solving practical reliability problems closely, and some evaluation methods have been proposed and applied to real problems.

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Mathematical studies about multi-state systems with totally ordered state spaces have been performed by many authors. See Barlow and Wu [2], Griffith [7], El-Neweihi et al. [18], Natvig [17], Ohi and Nishida [19–21], Ohi [23]. Huang et al. [8] have extended a binary state consecutive k-out-of-n system which is well observed in a real situation to multi-state case. Natvig [16], Lisnianski and Levitin [9], Lisnianski et al. [14] have summarised the work performed so far, and we find examples of practical applications of multi-state systems.

Levitin [10,12,13] have extensively applied the universal generating function (UGF) method for solving reliability problems of multi-state systems and showed its effectiveness. UGF method was first proposed by Ushakov [29,30] as a stochastic evaluation method of multi-state systems, and is especially thought to be effective for the stochastic analysis of a system hierarchically composed of modules like series-parallel or parallel-series systems. Ohi [26] has generally given stochastic upper and lower bounds for system's stochastic performances via modular decompositions, which are convenient for systems designers and analysists.

Examinations about the case of partially ordered states have been recently started. Levitin [11], from a practical point of view, has proposed new model of multi-state k-out-of-n system called multi-state vector-k-out-of-n system of which state spaces are assumed to be a set of vectors and then a special type of partially ordered set. Yu et al. [31], emphasising the case for the states not to be ordered, have proposed a model of multi-state system having partially ordered state spaces. Ohi [24,25] have been trying to build up a general theory of multi-state systems. The former paper has given an existence theory of series and parallel systems, series-parallel decomposition of multi-state systems, when the state spaces are lattice sets. The latter work has given a characterisation of a module by φ-equivalent relation under the lattice set assumption for the state spaces. Furthermore Ohi [27], a continuation of Ohi [26], shows upper and lower bounds for $\boldsymbol{P}\{\varphi \geqq s\}$ via a modular decomposition, the probability that the system's state is greater than or equals to s, which are extended to $\boldsymbol{P}\{\varphi \in A\}$ by Ohi [28], where A is an increasing subset of the system's state space, in other words, a subset of good states of the system, and the probability $\boldsymbol{P}$ is given on the product set of the state spaces of the components and is associated. But in these papers, only the stochastic bounds are shown without any proof and examples.

A module is a small scaled sub-system constructed of components which can be treated as a whole in the overall system. As the pipe line system of Natvig [16], a system is composed of many modules constructed hierarchically according to the size of and interrelation among them. An usual practical reliability evaluation of a system is performed by calculating first the reliabilities of the disjoint subsystems and then the reliability of the overall system. So the concept of a modular decomposition plays an important role in the reliability engineering. A modular decomposition of a binary state monotone system φ consists of two types of structure functions, structure functions of modules χ_i $(1 \le i \le m)$ and a structure function ψ of the system composed of the modules as components. Synergising a modular decomposition and stochastic bounds by the minimal path and cut sets of the system, we have the following inequalities for the reliability $h_\varphi(\boldsymbol{P})$ of the system φ.

$$h_\varphi(\boldsymbol{P}) \ge \left\{ \begin{array}{l} l_\psi(h_{\chi_1}(\boldsymbol{P}), \ldots, h_{\chi_m}(\boldsymbol{P})) \\ h_\psi(l_{\chi_1}(\boldsymbol{P}), \ldots, l_{\chi_m}(\boldsymbol{P})) \end{array} \right\} \ge l_\psi(l_{\chi_1}(\boldsymbol{P}), \ldots, l_{\chi_1}(\boldsymbol{P})) \ge l_\varphi(\boldsymbol{P}). \quad (1)$$

Refer to Bodin [6] and Barlow and Proschan [1]. h_φ is called the reliability function of the system φ, which corresponds the reliabilities of the components to the reliability of the system. The definition of it in the binary-state case is shown in Barlow and Proschan [1]. Mathematically, for a probability $\boldsymbol{P}$ on Ω_C, $h_\varphi(\boldsymbol{P})$ is the probability on the state space of the system S and is defined as

$$h_\varphi(\boldsymbol{P})(A) = \boldsymbol{P}(\varphi^{-1}(A)), \quad \text{for} \quad A \subseteqq S.$$

When the system is a binary-state system, it makes no confusion to understand $h_\varphi(\boldsymbol{P})$ to be $h_\varphi(\boldsymbol{P})(1)$ which means the probability that the system is normal, in other words, the reliability of the system. A generalisation of (1) to the case of totally ordered state spaces has been given by Ohi [26].

This paper gives more refined stochastic bounds for the reliability of a multi-state system in addition to perfect proofs of the theorems in Ohi [28]. Numerical examples are also given for a demonstration of the stochastic bounds. The concept of multi-state system of this paper is defined as the following.

Definition 1.1. A multi-state system composed of n components is defined to be a triplet (Ω_C, S, φ) satisfying the following conditions.

(i) $C = \{1, \ldots, n\}$ is a set of components.
(ii) $\Omega_i (i \in C)$ and S are finite partially ordered sets, denoting the state space of the ith component and the system, respectively. Each state space has the minimum and maximum elements which mean the perfect failure and the perfect functioning states, respectively. We use the following symbols for these special elements.

$$0_i = \min \Omega_i, \quad M_i = \max \Omega_i \quad (i \in C), \quad 0 = \min S, \quad M = \max S.$$

(iii) $\Omega_C = \prod_{i \in C} \Omega_i$ is the product ordered set of Ω_i $(i \in C)$. Each element $\boldsymbol{x} = (x_1, \ldots, x_n) \in \Omega_C$ is called a state vector, where x_i is the state of the ith component.
(iv) φ is a surjection from Ω_C to S, which is called a structure function and reflects a structure of the system.

A multi-state system (Ω_C, S, φ) is simply called a system φ, when there is no confusion.

Our probabilistic examination is for steady state or at some time slice, since time concept is not included. But, the Definition 1.1 is still a basis for examinations of stochastic dynamics of the system. When an Ω_i-valued stochastic process $\{ X_i(t), t \geqq 0 \}$ defined on a stochastic