Combustion and Flame 000 (2018) 1–14

Contents lists available at ScienceDirect

Combustion and Flame

journal homepage: www.elsevier.com/locate/combustflame

A diffusion-flame analog of forward smolder waves: (II) stability analysis

Zhanbin Lu[a,b,*]

[a] Institute of Applied Mathematics and Mechanics, Shanghai University, 149 Yan Chang Rd., Shanghai 200072, PR China
[b] Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai 200072, PR China

ARTICLE INFO

Article history:
Received 13 February 2018
Revised 4 July 2018
Accepted 5 July 2018
Available online xxx

Keywords:
Forward smolder wave
Diffusion flame
Heat loss
Fingering instability
Traveling wave instability

ABSTRACT

We proceed to examine the stability of the adiabatic and non-adiabatic structures of forward smolder waves elaborated in Part (I) of this series. The dispersion relation for adiabatic forward smolder waves with a reaction trailing structure turns out to take a form similar to that for premixed flames, thereby strengthening the analogy of the reaction trailing structure with the premixed flame regime of diffusion flames. According to the dispersion relation, corresponding to each Damköhler number there exists a marginal oxygen Lewis number, below which cellular instability occurs. In particular, similar to the Burke–Schumann limit of diffusion flames, the stoichiometric limit at infinite Damköhler number is unconditionally stable. Such unconditional stability is found to further extend to the entire Damköhler number range for adiabatic forward smolder waves with a reaction leading structure. Linear stability analysis of non-adiabatic forward smolder waves indicates that, for both reaction trailing and reaction leading structures, the low smolder temperature (or high reactant leakage) solution branch is physically unrealistic, whereas on the high smolder temperature (or low reactant leakage) branch different kinds of instabilities may develop near the quenching limit. Under a fixed Damköhler number, the range of the heat loss coefficient corresponding to these instabilities shows a trend to grow with decreasing oxygen Lewis number. 2-D time-dependent numerical simulations of unstable non-adiabatic forward smolder waves confirm that fingering or cellular instability occurs exclusively for the reaction trailing structure, whereas traveling wave instability prevails for the reaction leading structure. A comparison is made between the current stability analysis results of non-adiabatic forward smolder waves and results from a concurrent flame spread experiment. Agreement is achieved not only on the existence of reaction front instabilities near the quenching limit, but also on the conditions determining the type of these instabilities.

1. Introduction

Similar to flames, smoldering processes may suffer various kinds of instabilities. Early experimental evidence of such instabilities comes from the filtrational configuration [1], for which it was found that pulsating burning fronts could develop in a porous medium filled with oil. Since then, many efforts have been made to examine the stability of combustion fronts in the filtrational context, by employing various kinds of models of smoldering combustion [2–8]. Most of these studies focused on the diffusive-thermal aspects of the instability, and besides the pulsating propagation mode, cellular or fingering instability and traveling wave instability of the combustion fronts have also been identified under different parametric conditions. Aldushin and Matkowsky [9] and Brailovsky et al. [10] examined the hydrodynamic instability of filtrational combustion waves, showing that fingered patterns generally result after the onset of the instability. However, different from fingered patterns resulting from diffusive-thermal instability of the combustion fronts, fingers resulting from hydrodynamic instability are due to nonuniform reduction of flow resistance along the wave front.

Because of the difficulties with observation and measurement involved in the filtrational configuration, most experimental studies addressing the stability of smoldering combustion employed the fuel layer configuration, where the oxidizing gas is either driven by natural convection or forced to sweep over the surface of a solid fuel layer. Depending on the fuel material properties and the atmospheric conditions, the combustion reaction taking place in the spreading reaction front can be either heterogeneous or homogeneous, the latter mode defining the classical flame spread problem [11,12]. The stability properties of the reaction front, however, are presumed not to be quite sensitive to the specific mode of the reaction. Hence, from the perspective of front stability,

* Corresponding author at: Institute of Applied Mathematics and Mechanics, Shanghai University, 149 Yan Chang Rd., Shanghai 200072, PR China.
E-mail addresses: zblu@i.shu.edu.cn, zblu@shu.edu.cn

https://doi.org/10.1016/j.combustflame.2018.07.006
0010-2180/

experimental results of smolder spread and flame spread can generally be compared and discussed within the same context.

In an experiment addressing natural counterflow flame spread over thin paper strips, Zhang et al. [13] showed that reducing the oxygen Lewis number eventually led to the onset of cellularity of the spreading diffusion flame front. Likewise, in a microgravity smolder experiment, Olson et al. [14] observed the formation of fingered burned-out patterns on the surface of a thin cellulosic fuel, over which a uniform, opposing air flow was applied. Similar fingering instability of reverse smolder waves was also identified by Zik et al. [15–17] in experiments conducted in a narrow channel apparatus, which was employed to simulate the microgravity environment by suppressing vertical buoyant convection. Subsequent experiments of opposed-flow flame spread over thin cellulosic fuels conducted in narrow channels [18,19] revealed similar fingered patterns, thereby corroborating the above-mentioned presumption about the insensitivity of the stability properties to the reaction mode. Various modeling efforts [7,20–26] have been made to gain insight into the underlying physical mechanisms of these experimental findings. In this regard, a consensus is that the cellular or fingered patterns observed in the experiments were nothing but a manifestation of diffusive-thermal instability of the reaction fronts, which is essentially similar to the diffusive-thermal instability of premixed or diffusion flames.

Most of the studies mentioned above on smolder waves or flame spread have been restricted to the reverse configuration, whereas studies addressing the stability of forward smolder waves or concurrent flame spread are relatively rare in the literature. In a numerical study examining the diffusive-thermal instability of forward smolder waves, Lu and Dong [27] found that, similar to the reverse configuration, fingering instability can develop near the quenching limit when heat loss effects are incorporated. This numerical prediction has recently been verified by a concurrent flame spread experiment conducted in a narrow channel [28], which showed that, for oxygen concentrations below a certain critical value, fingered patterns develop on the surface of a thin cellulosic fuel when the oxidizer gas flow velocity is sufficiently close to the quenching limit. When the oxygen concentration is increased beyond the critical value, however, near the quenching limit the fingering instability is superseded by a traveling wave instability, which is characterized by a transverse creeping motion of the flamelets along the unburned fuel edge.

As discussed in Part (I) of this series [29], forward smolder waves are structurally analogous to diffusion flames, so an understanding of the stability properties of diffusion flames shall be helpful to understand the stability of forward smolder waves. In an experimental study on diffusive-thermal instability of slot burner diffusion flames, Chen et al. [30] observed the formation of cellular flame structures under conditions where the extinction limits are approached and the effective Lewis number is sufficiently small. Subsequently, extensive analytical studies have been carried out to examine the stability of diffusion flames [31–38], and in this regard both cellular and pulsating instabilities have been identified near the low-Damköhler-number blowoff limit. When heat loss effects are present, near the large-Damköhler-number quenching limit, pulsating instability was identified by Sohn et al. [39] and Miklavcic et al. [40] through an examination of model diffusion flames. A detailed account of diffusion flame instability is available in a review by Matalon [41].

The above literature survey indicates that, for adiabatic forward smolder waves, a comprehensive stability analysis that covers both the reaction trailing and reaction leading structures over the entire range of Damköhler number and oxygen Lewis number is still lacking. As for the stability of non-adiabatic structures, Ref. [27] considered only the reaction *trailing* structure under one fixed Lewis number, and the heat loss coefficient was assumed to be dependent on the Damköhler number. Although the predicted fingering instability was verified by the concurrent flame spread experiment reported in Ref. [28], the traveling wave instability identified in the experiment and the transition mechanism between these two distinct instabilities call for further investigation. To fill these gaps, in this work we present a comprehensive stability analysis of forward smolder waves, covering both adiabatic and non-adiabatic cases, and for each case addressing both the reaction trailing and reaction leading structures. For the adiabatic case, stability is examined over the entire range of Damköhler number and oxygen Lewis number, whereas for the non-adiabatic case, attention is primarily focused on the vicinity of the quenching limit, with the heat loss coefficient and the oxygen Lewis number as two primary control parameters. Following Part (I) [29], the structural analogy of forward smolder waves with diffusion flames is exploited in the interpretation of the stability analysis results.

To render the presentation self-contained, we start in Section 2 with a brief summary of the formulation and the 1-D adiabatic steady solutions obtained from Part (I). This is followed by a linear stability analysis of adiabatic and non-adiabatic forward smolder waves in Sections 3 and 4, respectively. Section 5 is devoted to direct time-dependent numerical simulations of unstable non-adiabatic structures, and in Section 6 a comparison is made between the current numerical results and the experimental results reported in Ref. [28]. Finally, conclusions are given in Section 7.

2. Summary of the governing equations and 1-D adiabatic steady solutions

This section summarizes the equations governing forward smolder combustion and the corresponding 1-D adiabatic steady solutions. For further details regarding the model assumptions, formulation, and the derivation process of the steady solutions, the reader is referred to Part (I) of this series [29].

2.1. Governing equations and boundary conditions

In the reference frame that is kept stationary with respect to the solid phase, the dimensionless equations governing forward smolder combustion take the form

$$
\begin{aligned}
\frac{\partial T}{\partial t}+\frac{\partial T}{\partial x} &= \nabla^2 T+\Omega-H(T-T_\mathrm{f}),\\
\frac{\partial Y}{\partial t} &= -\Omega = -\mathrm{Da}\,XY\exp\left\{-\frac{\theta}{T}\right\},\\
r_\rho\frac{\partial X}{\partial t}+\frac{\partial X}{\partial x} &= \frac{1}{\mathrm{Le}}\nabla^2 X-\Omega,
\end{aligned}
\tag{1}
$$

where t represents time, x is the spatial coordinate measured along the same direction as the smolder wave propagation, T, Y, and X represent, respectively, the dimensionless temperature and the normalized fuel and oxygen mass fractions, Ω is the dimensionless reaction rate, H is the dimensionless heat loss coefficient, Da is the Damköhler number, Le is the oxygen Lewis number, θ and T_f are, respectively, the dimensionless activation and ambient temperature, and r_ρ is the gas-to-solid density ratio; see also the Nomenclature of Part (I) [29] for the definition of the variables and parameters.

In the presence of heat loss, Eq. (1) is subject to the following boundary conditions:

$$
\begin{aligned}
&\text{as } x\to-\infty,\quad T\to T_\mathrm{f},\quad \frac{\partial Y}{\partial x}\to 0,\quad X\to X_\mathrm{f},\\
&\text{as } x\to+\infty,\quad T\to T_\mathrm{f},\quad Y\to 1,\quad \frac{\partial X}{\partial x}\to 0,
\end{aligned}
\tag{2}
$$

where X_f is the normalized fresh oxygen mass fraction at far upstream.

Please cite this article as: Z. Lu, A diffusion-flame analog of forward smolder waves: (II) stability analysis, Combustion and Flame (2018), https://doi.org/10.1016/j.combustflame.2018.07.006