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Free and forced large amplitude vibrations of periodically inhomogeneous slender beams

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ABSTRACT

Considered are free and forced transverse vibrations of slender periodic beams of finite length. It is assumed that the vibration amplitude is of the order of cross-section dimensions, still much smaller than the beam length. An averaged non-asymptotic model is proposed as a tool in analysis. The description is based on the tolerance approach to averaging of differential operators, using the concept of weakly slowly-varying function. The resulting differential equations with constant coefficients involve the effect of periodicity cell length. The model is verified by comparison of linear frequencies and mode shapes with Finite Element Method results, and then applied in analysis of free and forced vibrations of beam with variable cross-section. The method employed in obtaining the solution involves Galerkin orthogonalization and Runge–Kutta (RKF45) method. The results of nonlinear vibrations analysis are presented by backbone and amplitude-frequency response curves, time series, Poincare sections and bifurcation diagrams.

1. Introduction

Structural members of periodically distributed physical properties represent a special group of inhomogeneous structures. On the local level they are formed as usually piecewise continuous structures by joining together identical elements in organized manner. On the global level, such structures exhibit some effective properties and, especially in cases of 2-D or 3-D elements, can be treated as anisotropic or orthotropic.

Depending on the level and scope of investigation, various properties of periodic structures are revealed and emphasized. Considering static problems, such as bending or stability issues, densely ribbed plate and shell panels exhibit favourable weight to stiffness ratio [2,11], which confirms the advantages of the use of periodically arranged stiffeners in lightweight structures. On the other hand, the interest in the use of auxetic materials is growing, supported in observations of their unique properties like Poisson's ratio sign dependent on the load direction, cf. [20]. In the domain of structural dynamics, the most interesting features of periodic structures refer to their wave filtering and vibration attenuation properties [6,8,25]. Most of investigation effort is directed towards analysis of wave propagation in periodic 1-D and 2-D structures [23]. Usually, the investigation is limited to beams of infinite length, which allows analysis of a single periodicity cell and employing the Bloch–Floquet theorem [24].

The methods applied in analysis of such structures can generally be divided into two groups: discretization and averaging methods. The direct application of discretization methods leads to models of large number of degrees of freedom. Such approach, supported with optimization algorithms, was successfully employed in in order to maximize the frequency band gaps in finite 1-D and 2-D structures [9,16]. In [26] high frequency dynamics of an aluminium Timoshenko

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periodic beam is investigated through numerical simulations and experimental measurements. The source of the beam periodicity is the presence of a number of appropriately arranged drill-holes. Between the averaging methods, the most widespread is asymptotic homogenization [13] which was, amongst many other works, applied in [7] to estimate upper and lower bounds for low-order frequencies of composite periodic beam. In [10] transition from a 3-D elasticity problem with initial stresses to beam theory is made, making use of two-scale homogenization. Equivalent representation of periodically variable cross-section based on simple yet well-established idea was applied in [25] in comprehensive study of static and dynamic problems of beams, taking into account the non-effective transition regions. Analysis of linear eigenfrequencies and mode shapes of beams with continuously varying cross-section was conducted in [18] using the method of varying amplitudes.

When geometrically nonlinear dynamics problems of composite structural elements are considered, it is usually done for layered members [1]. The formerly mentioned method of varying amplitudes was also applied in [19] to investigate the effects of weak geometrical and material nonlinearity on wave dispersion relations for beams with smoothly variable cross-section. The study was also supported by experimental results.

The tolerance modelling technique [22] was applied in analysis of various thermomechanical problems of periodic and functionally graded structures, such as geometrically nonlinear static problems of densely ribbed plates [2], linear dynamics of periodic beams under moving load [15] or heat transfer problems [17]. The concept of weakly slowly varying function in the context of averaging differential operators was introduced in analysis of periodically stiffened shells [21]. In papers [3,4], a considerably simplified versions of proposed beam model were described and applied in analysis of uniform stiffness beams with lumped masses attached. In the previous studies, the derivation was based on assumption that all the unknown functions are slowly varying in x, which leads to differential equations of lower order. Thus, previously proposed models are valid only in a certain range of frequencies. To be more precise, they give correct results in the vicinity of frequency band gaps. Moreover, the present paper delivers the variational derivation of natural boundary conditions involving nonlinear terms and rotational inertia. Thus, the described solution procedure makes it possible to analyse vibrations in the whole range of frequencies within applied theory of slender beams.

The main of this paper aims are: to obtain an non-asymptotic averaged model of periodic beam taking into account moderately large deflections, and to demonstrate the applicability of proposed approach in analysis of free and forced vibrations. The paper is organized as follows. Section 2 contains a brief reminder of general principles and equations of geometrically nonlinear beam theory with emphasis on the main problems of application. In Section 3 the fundamental assumptions of the tolerance approach are introduced, and the averaged models of beam dynamics are derived. The next section contains a brief overview of the methodology for obtaining solutions of the averaged equations. Section 5 is devoted to analysis of special cases. The considered problem is stated, then more details corresponding to the solution method are given. The proposed approach is then justified by comparison of results obtained from linear eigenproblem analysis with those from the full finite element model of the beam. Two last subsections present the results of free and forced vibration analysis in the moderately amplitude vibrations range. The closing remarks and future work are given in Section 6.

2. The governing equations

A fragment of a typical periodic beam is depicted in Fig. 1. In an orthogonal Cartesian coordinate system $Oxyz$, the Ox axis coincides with the axis of the beam, the cross section of the beam is symmetric with respect to the plane of the load Oxz, the load acts in the direction of the axis Oz. The problem can be treated as one-dimensional, so that we define the region occupied by the beam as $\Omega \equiv [0, L]$, L stands for the beam length. The beam is made of linearly elastic material of Young modulus $E(x)$ and mass density $\rho(x)$, its cross-section characteristics are the area $A(x)$ and moment of inertia $J(x)$. The beam can bilaterally interact with a periodic viscoelastic subsoil, the elastic and the damping coefficient of which are $k = k(x)$ and $c = c(x)$. The beam is assumed to be made of many repetitive small elements, called periodicity cells, each of which is defined as $\Delta \equiv [-l/2, l/2]$, where $l \ll L$ is the length of the cell and named the mesostructure parameter.

Our considerations are based on the Rayleigh theory of beams with von Kármán type nonlinearity. We also take into consideration initial elongation or shortening of the beam axis. For the subsoil we assume the Kelvin–Voight model [14]. The effect of axial inertia is neglected, for we are interested in the transverse vibrations only. Let $w = w(x, t)$ be the transverse deflection, $u_0 = u_0(x, t)$ the longitudinal displacement, $EA = E(x)A(x)$ and $EJ = E(x)J(x)$ tensile and flexural stiffness, $\mu = \rho(x)A(x)$ and $\vartheta = \rho(x)J(x)$ mass and rotational moment of inertia per unit length and $q = q(x, t)$ the transverse load. Let $\partial^k = \partial^k/\partial x_k$ be the kth derivative of a function with respect to the x coordinate and $\partial F \equiv F$, overdot stands for the derivative with respect to time.

The fundamental Euler–Bernoulli theory assumption is that the lines perpendicular to the beam axis stay straight and perpendicular to this axis after the deformation. There are three main sources of nonlinearity in structural members behaviour: (1) nonlinear stress–strain relation of the material, (2) nonlinear curvature, (3) the so-called von Kármán nonline-

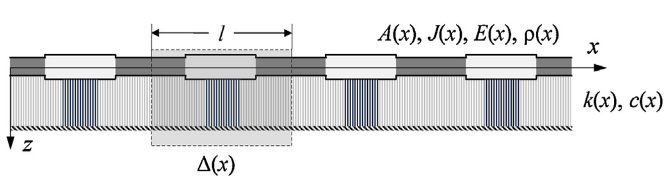

Fig. 1 – A fragment of periodically inhomogeneous beam, Δ(x) – a periodicity cell.