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Survey of precedence relationships: Classification and algorithms

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ABSTRACT

Precedence relationships used in project planning mean for most professionals the traditional Start-to-Start-z (SSz), Finish-to-Start-z (FSz), Finish-to-Finish-z (FFz) and Start-to-Finish-z (SFz) relationships where *z* stands for the minimal necessary duration between the defined endpoints (Start or Finish) of the activities. These relationships have been serving professionals for more than 50 years, and there is not much visible effort for further developing them, despite some well-established critiques on the modeling capability of the Precedence Diagramming Method (PDM). The purpose of this research is: a) gathering those well-known and lesser-known developments of logical relationships that can be used for modeling some, so far, un-modelable problems b) classifying them by using a classification scheme that has been developed for this purpose and c) developing algorithms for time analysis when missing. The following earlier developments are discussed: maximal precedence relationships, point-to-point precedence relationships, continuous precedence relationships, relationships with AND/OR logical switches and bidirectional precedence relations. The classification shows that 24 types of logical relationships exist, but that algorithms exist only in four types, and are missing in twenty cases. The missing algorithms are provided here. The main contributions to the Body of Knowledge are: a) providing a classification scheme for precedence relationships b) definition of 24 precedence relationships based on the classification categories c) developing the missing time analysis algorithms for twenty cases d) presenting a single unified algorithm that handles all the 24 types of precedence relationships.

1. Introduction

The Precedence Diagramming Method (PDM) is undoubtedly the prevailing project scheduling technique of our time. Its fundamental cornerstones are the activities that are assumed to be linear and the precedence relationships used for describing the technological or organizational logic between activities by connecting the end-point of the activities with the corresponding time gap that defines the necessary minimal time between the connected points. A further assumption regarding the PDM construct is the existence of one start activity (an activity without preceding activities) and one finish activity (an activity without successor activities). These activities represent the start and finish of the project in time. A frequently used graphical interpretation of this construct is a directed graph called activity-on-node network (AoN). Here, nodes represent activities, and directed arcs, with the attached information regarding the connected points and the lead/lag times, represent logical relationships between activities. Loops in this graph are not allowed. It is assumed that at least one path leads from the start node to all other nodes, and at least one path leads from all the nodes to the finish node. In the rest of this paper, this structure is referred to as *traditional* PDM network. (See Section 2).

There are many existing practical problems that are hard or impossible to model with the use of traditional precedence relationships (SS, FF, SF and FS). Some of them are presented below:

a) Prescribing maximal temporal distance. Propping of a hole (act. *C*) can start right after the finish of the excavation (act. ***A***), but it should start within two days of finishing the earthwork to avoid the collapse of the side walls. Material for propping comes from other activity. (act. ***B***) The prescribed maximum 2-day duration between the finish of ***A*** and the start of ***C*** is impossible to model with the traditional relationships. Using traditional precedence relationships, it is possible – if ***B*** is scheduled later than *A* – that C cannot start within two days after the finish of A and the sidewalls of the hole collapse.
b) Overlapping. The excavation of a long ditch can be followed by pipe laying in such a way that the time distance must be at least two days or the spatial distance - using work units - has to be at least 50 m. Solutions using traditional precedence relations based on the combination of SS and FF relations are flawed because these relationships control only the endpoints of the activities. Anything can happen between the start and finish of the activities in case of

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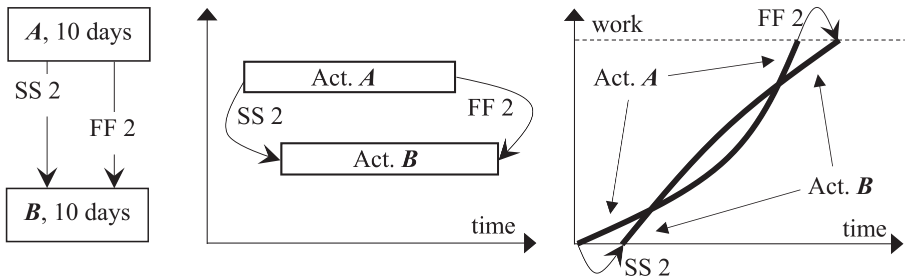

Fig. 1. Flawed modeling of overlapping using SS, FF relationships in case of nonlinear activities.

nonlinear activities; however, the network does not tell us about the possible conflicting situations either during planning or during control if SS and FF relationships are satisfied since these relationships control only the endpoints of the activities. A situation like this is shown in Fig. 1.

c) Conditional relations. Concreting is being carried out at three different places using the same new mix. Sampling for quality control can be carried out at any of the three locations and must be taken as soon as possible, after finishing the work at any of the sites.

d) Sequencing. The same machine is necessary for executing three different activities. Activities can be carried out in parallel, but only one machine is available for these activities, which makes parallel execution impossible. The order of execution is immaterial; however, the project must be finished as soon as possible.

The problems described above are hard or impossible to model using traditional precedence relations. These problems are either skipped during planning or imperfectly modeled based on some combinations of the traditional end-point precedence relationships. Any of the above practices leads to a schedule that contains missing or bad logic, the consequence of which is an incorrect schedule. Consequently, the need for expanding the modeling capabilities of the PDM technique is apparent. Therefore, one of the main goals of this research is to define those precedence relationships that allow the problems above to be modeled. Before defining the goals of this research in detail, a literature review is presented.

2. Literature review

The PDM technique has been developed parallel and independently from each other in the USA and Europe. One of the most influencing American pioneers of the technique was John Fondahl, who realized that the drawing of activity-on-arrow (AoA) networks is almost impossible in the case of larger networks. He has proved his recognition through a documented experiment ([1], Appendix H). To overcome the graphical presentation problem, he proposed the use of AoN representation instead of AoA representation. With this, he created the opportunity to attach different information to the arcs; first, the necessary time gaps between activities, later the information about which endpoints are connected. This criticism, regarding the problems of AoA portrayal, was later theoretically supported by Krishnamoorty & Deon [2] who proved that graphical representation aiming to draw an AoA network using the minimal number of dummy activities is a non-deterministic polynomial-time complete (NP-complete) task.

According to the reminiscences of Fondahl [3], the term “precedence diagramming” originates from the IBM team who came up with a project scheduling and controlling application called Project Control System (PCS) in 1964 for the IBM 1440 computer. Unfortunately, this author could not trace the user manual of this application. However, the term “precedence diagramming” is mentioned among the features of PCS in an IBM product catalogue for the IBM 1440 computer. According to it: “Planning accomplished through conventional arrow diagramming or the new, highly efficient and easier to use precedence diagramming” (IBM 1968, p. 33). Also, a user manual of PCS for IBM 1310 from 1967 clearly describes the minimal precedence relationships (IBM1310 1967). That is why recently applied precedence relationships with minimal lead/lag will be referred to as *traditional* precedence relationships. The term endpoint relationships will also be used, alluding to that traditional precedence relationships connect the endpoints of the activities. The term generalized precedence relation (GPR) is also used for these relationships, especially in operational research related literature.

Roy, the European pioneer of PDM, has defined an AoN network where SSz relationships were applied. His technique was called the Metra Potential Method (MPM) and was so advanced that it made possible the application of SS relation using a maximal time gap between the start points [4]. It is interesting to note that linearity of activities has been a prerequisite for PDM activities, but this was not a requirement in Critical Path Method (CPM). The reason for this is twofold. On the one hand, CPM describes a logical system where logically tied activities are consecutively following each other, while PDM precedence relationships can be used for defining overlapping. SS and FF relationships can be used to describe overlapping only if linearity is a prerequisite. If not, then the situation shown in Fig. 1 can arise. Fig. 1 shows the same two overlapping activities using network, Gantt and Linear Scheduling Method (LSM) portrayal. It can be seen that both SS and FF relationships are satisfied, however a conflicting situation can arise if activities are not linear. FF and SS combination can be used for modeling overlapping if and only if linearity of activities are assumed. (See Fig. 1) On the other hand, time analysis in the case of dangling activities (activities without preceding relationships to the start point, or without successor relationships from the finish point) can lead to minus infinite early start dates and infinite late finish dates if splitting is allowed. However, this second issue can be solved by prudent planning that does not allow danglers to remain in the network.

Modeling of different seemingly non-modelable practical problems is always an issue among planners [5]. Initially, solutions were sought using a combination of traditional precedence relationships; later, researchers devoted more time to extending the modeling capabilities of the technique by offering new precedence relationships as follows:

- maximal precedence relationships, (extension of Roy's work)
- point-to-point (PtP) precedence relationships
- continuous precedence relationships
- AND/OR logical switches on precedence relationships
- bidirectional precedence relationship

Historical reviews for these lesser-known relationships are given in Section 4, where the solutions of the problems listed in Section 1 are also provided.

3. The research goal

The aim of this research is to provide solutions for some practical problems by the introduction of new precedence relationships, classifying them to create a unified frame for developing the algorithms