ARTICLE IN PRESS

Journal of Sound and Vibration 411 (2017) ■■■–■■■

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A novel hybrid surrogate model and its application on a mechanical system subjected to friction-induced vibration

E. Denimal [a,b], L. Nechak [a], J-J. Sinou [a,c,*], S. Nacivet [b]

[a] Laboratoire de Tribologie et Dynamique des Systèmes, UMR CNRS 5513, École Centrale de Lyon, 36 avenue Guy de Collongue, 69134 Écully Cedex, France
[b] PSA Peugeot Citroën, Centre technique de la Garenne Colombes, 18 rue des Fauvelles, 92250 La Garenne Colombes, France
[c] Institut Universitaire de France, 75005 Paris, France

ARTICLE INFO

Article history:
Received 13 March 2017
Received in revised form
1 August 2017
Accepted 4 August 2017
Handling Editor: A.V. Metrikine

Keywords:
Stability
Complex eigenvalue analysis
Mode coupling
Friction
Meta-modeling
Kriging
Generalized polynomial chaos
Uncertainty

ABSTRACT

A new hybrid meta-modeling approach is proposed and developed for the propagating and quantifying of uncertainties in friction-induced instabilities. According to the available knowledge about design parameters, the associated uncertainty can be described by using different models. Hence, hybrid meta-models prove to be necessary to simultaneously treat different uncertainty models. So, this study presents a novel hybrid meta-model issued from the associating of the kriging formalism together with the generalized polynomial chaos for the prediction of friction-induced instabilities submitted to interval and probabilistic uncertainties. Its assessing through the considering of a friction system, reveals suitable accuracy about the estimating of the dispersion of the occurrences of instabilities. Moreover, it offers a promising alternative to the prohibitive Monte-Carlo/scanning based methods that are usually used for the same task.

1. Introduction

The uncertainty propagation, as well as the uncertainty quantification, in friction-induced instabilities are central steps helping for the setting up a robust design of mechanical systems. They are required in view of the non-negligible sensitivity of the stability property and, more generally, the dynamic behavior of self-excited frictional system to the variations of design parameters. These facts have already been highlighted by several experimental and numerical studies. For example Bultlin and Woodhouse [1,2] demonstrated that friction-coupled systems are intrinsically sensitive to parameter variations and uncertainties. Oberst and Lai [3] also proposed an experimental approach to the statistical analysis of friction-induced instabilities in brake systems. Also, even if Culla and Massi [4] investigated contact instability under the influence of uncertainties via Monte Carlo simulation (MCS), it is now admitted that carrying out stability and non-linear self-excited friction-induced vibrations with uncertainties by using the conventional methods like scanning and/or Monte Carlo techniques is prohibitive and too costly in terms of computation time. Indeed, the use

---

[*] Corresponding author at: Laboratoire de Tribologie et Dynamique des Systèmes, UMR CNRS 5513, École Centrale de Lyon, 36 avenue Guy de Collongue, 69134 Écully Cedex, France.

E-mail addresses: enora.denimal@doctorant.ec-lyon.fr (E. Denimal), lyes.nechak@ec-lyon.fr (L. Nechak), jean-jacques.sinou@ec-lyon.fr (J-J. Sinou), samuel.nacivet@mpsa.com (S. Nacivet).

http://dx.doi.org/10.1016/j.jsv.2017.08.005

Please cite this article as: E. Denimal, et al., A novel hybrid surrogate model and its application on a mechanical system subjected to friction-induced vibration, Journal of Sound and Vibration (2017), http://dx.doi.org/10.1016/j.jsv.2017.08.005

of such techniques requires high numbers of samples within the design space to ensure suitable accuracies and confidences of results.

Thus, one of the most important challenges today is to be able to considerably reduce the number of calculations to be carried out in order to predict friction-induced instabilities with uncertainties. In order to overcome this problem, strategies based on the searching for surrogate models, named 'meta-models', have been developed. A wide interest has been given to probabilistic meta-models and more particularly to the Generalized Polynomial Chaos (GPC) expansion to predict random friction-induced instabilities [5–7] when uncertain parameters are described by Probability Density Functions (PDF). When the probability distributions of uncertain parameters are not defined unambiguously (due to limited data of system parameters or the great number of experimental samples needed to perform statistics for example), it becomes more suitable to use non-probabilistic approaches including fuzzy methods [8] or interval analysis [9]. Another type of meta-model is based on kriging formalism [10,11]. Such a meta-model approximates a parameter-dependent function by using a small number of parameter samples generated according to a predefined experimental design. Recently, kriging has been used successfully for the prediction of friction-induced instabilities [12–15].

Nowadays in the field of friction-induced instabilities, efforts have to be dedicated to the need for combining probabilistic uncertainty, due to the variability knowledge of some parameters, and intervals or fuzzy uncertainty, due to missing information or missing precision of other parameters. So, it becomes necessary to develop hybrid meta-models that are able to simultaneously propagate and quantify these uncertainties. In this context, mixed uncertainty methods have also been proposed by several researchers [9,16]. Otherwise, numerous software packages have been proposed to deal with meta-modeling for uncertainty propagation as well as for robust design optimization [17,18].

In order to meet the challenge of combining aspects of interval analysis and probability theory in a suitable way for calculations of friction-induced instabilities, this paper proposes an hybrid meta-model based on the GPC expansion and the kriging formalism. Such an association has recently been proposed by Kersaudy et al. [19]. The main idea of their work consists in the replacing of the regressive part in the kriging meta-model by a GPC expansion. The approach presented here is quite different since it consists in the modeling of the random dimension with the GPC expansion and the dependence on the interval variables with the kriging. The determining of the hybrid meta-model is then turned into the problem of the evaluation of the coefficients of the GPC expansion by using kriging meta-models.

The paper is organized as follows. First, the kriging and GPC formalisms are presented in Section 2. Then the description of the mathematical formulation of the proposed hybrid meta-model is discussed. The mechanical model under study is briefly described in Section 3. Finally, efficiency and accuracy of the hybrid uncertainty propagation and quantification on the stability behaviour of the mechanical system under study is evaluated and commented.

2. Meta-modelling methods

This section is devoted to the description of the proposed hybrid meta-model issued from the association of the GPC expansion together with the kriging meta-model. The latter are first separately presented. The shared property of both meta-models is related to their non-intrusive calculation (i.e. there is no need to modify the system's equations). Only a small size set of learning data defined by a set of input/output samples generated according to a predefined plan of experiments, is required for the obtaining of the two meta-models and so the hybrid one.

2.1. Mathematical formulation of kriging

Let y be a function in the vector of parameter $\mathbf{x} \in \mathbb{R}^l$, l being the dimension of the design space. In the present study, y represents the real or the imaginary part of the complex eigenvalues of the mechanical system subjected to friction-induced vibrations (see Sections 3 and 4 for more details).

Based on kriging theory, the parameter-dependent function y can be approximated by Y defined from a regression model characterizing the average behavior of y, one of which the dispersions are characterized by a spatial correlation function that measures how points in the design space are spatially close one to each other. This is formalized as follows:

$$Y(\mathbf{x}) = \mathbf{g}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\beta} + Z(\mathbf{x}) \tag{1}$$

where $\mathbf{g}(\mathbf{x})$ is the regression term composed of a q basis polynomial functions $\mathbf{g}_i$ weighted by regression parameters $\boldsymbol{\beta}_i$. The second term $Z(\cdot)$ is the realization of a zero-mean Gaussian process with a covariance matrix given by:

$$\mathbb{E}[Z(\mathbf{s}), Z(\mathbf{x})] = \sigma^2\mathcal{R}(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x}) \tag{2}$$

where σ^2 is the process variance and $\mathcal{R}(\boldsymbol{\theta}, \mathbf{s}, \mathbf{x}) \in [0, 1]$ is the spacial correlation function with the scaling parameter $\boldsymbol{\theta} \in \mathbb{R}^l$ while $\mathbb{E}[\cdot]$ denotes the expectation operator. The correlation function is monotone and its construction is such that two identical points have a unitary correlation and two infinitely separated points have a zero correlation. Different choices of correlation functions are possible [17]. To construct a kriging model, the regression parameters $\boldsymbol{\beta}$, the process variance σ and the scaling parameter $\boldsymbol{\theta}$ have to be determined. This construction is strongly related to the experimental design set $\mathbf{S} = \left(\mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(N)}\right)$ of N sample points of the design space and the associated image $\mathbf{y}_S = \left(y(\mathbf{s}^{(1)}), \ldots, y(\mathbf{s}^{(N)})\right)$. The parameter $\boldsymbol{\theta}$ is

Please cite this article as: E. Denimal, et al., A novel hybrid surrogate model and its application on a mechanical system subjected to friction-induced vibration, Journal of Sound and Vibration (2017), http://dx.doi.org/10.1016/j.jsv.2017.08.005