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Scattering of low frequency sound by fluid and solid cylinders

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Wave scattering by objects is typically studied for plane incident waves. However, full Green's functions are necessary for problems where the separation of the scatterer from interfaces, sources, or other scatterers is comparable to the dimensions of the scatterer itself. In this paper, the two-dimensional problem of scattering of monochromatic cylindrical waves by an infinite cylinder embedded in a homogeneous fluid is considered. Fluid and solid cylinders are studied, and soft and hard cylinders are revisited. The exact solutions for the Green's functions are expressed as an infinite series of cylindrical functions with complex amplitudes determined by the acoustic boundary conditions at the surface of the cylinder. Here, we derive closed-form asymptotics for the scattered field in the Rayleigh scattering regime where radius of the cylinder is small compared to the wavelength. The scattered wave approximation is valid for arbitrary source and observation point positions outside the scatterer and is expressed as a sum of fields due to three linear image sources. When the source or receiver is located sufficiently far from the cylinder, the new uniform asymptotic solutions reduce to well-known results for plane-wave scattering. Image source solutions were anticipated due to classically studied electrostatic analog problems involving dielectric cylinders. Image source representation offers physical insights into the scattering physics and suggests simple analytic solutions for scattering by objects near interfaces and within waveguides.

1. Introduction

Most typically sound scattering by circular cylinders has been studied for plane incident waves. The exact solution of the problem of a plane wave scattered by an infinite cylinder was obtained by Lowan et al. for fluid [1] and Faran for solid [2] targets. The solutions are given in terms of an infinite series of the products of Bessel functions. In a multitude of problems, the plane wave solution is not sufficient and the full Green's function of the problem, rather than the plane-wave and far-field approximations, becomes necessary [3–7]. The acoustic field due to a linear source parallel to the axis of a cylinder may be viewed as a two-dimensional (2D) Green's function. The Green's function describes scattering of an incident cylindrical wave by the cylinder. A recent example of application of 2D Green's functions is development of a diffraction-based technique for passive suppression of radiation of low-frequency noise in underwater waveguides [8]. In addition to situations where a sound source is located in the near field of the scatterer, knowledge of the Green's function is required when the separation distance from the scatterer to an interface or other obstacles is comparable to the linear dimensions of the scatterer itself [9–13].

Wave scattering by objects that are small relative to the wavelength is commonly referred to as Rayleigh scattering. Rayleigh scattering is studied in various branches of wave physics [14–16] and is of particular practical interest in acoustics

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[17–19], especially in underwater acoustics [20–22]. The distinct physics of low-frequency scattering allows for a considerable reduction in complexity in the mathematical description of the Green's function. Derivation of simple and accurate closed-form approximations of the 2D Green's function at scattering from a circular cylinder within the Rayleigh regime is the goal of this work.

In this paper, we consider Rayleigh scattering of a cylindrical wave by a fluid or solid cylindrical obstacle. Recently an analytic approach was developed to approximate the acoustic Green's function in the Rayleigh regime. The approach utilized the method of matched asymptotic expansions to establish uniform asymptotic solutions for the diffraction of a monochromatic spherical sound wave by small soft, hard, impedance, and homogeneous fluid spherical targets [23,24]. The method was also applied to establish solutions for monochromatic cylindrical wave scattering by infinite soft, hard, and impedance cylindrical targets [25]. These solutions offer a mathematically simple and intuitive representation of the scattered field as the field due to “image sources” located within the object.

Image source solutions were anticipated based on classical elementary solutions to problems in electrostatics and magnetostatics involving spheres and cylinders in the presence of point and line charges, respectively [26–30]. Of the static solutions, the best-known is Lord Kelvin's solution [31] for the electric field of a point charge in the presence of a grounded perfectly electrically conducting sphere. Here, we extend the asymptotic solutions for scattering of cylindrical acoustic waves, which were previously established for the soft, impedance, and hard cylinders, to the more realistic cases of fluid and solid cylindrical obstacles. Here, soft and hard boundary conditions correspond to pressure release and acoustically rigid surfaces, respectively, and the impedance boundary condition represents a linear combination of both soft and hard boundary conditions for modeling of more complex material scatterers (see Eqs. (4)–(6) from Ref. [25]). Importantly, none of which accounts for wave propagation within the target and as a result only provide first-order approximations to the scattering physics that are described by real fluid and solid material targets. The new uniform asymptotic solutions for fluid and solid scatterers are valid in the Rayleigh regime and describe the scattered field everywhere outside of the cylinder for arbitrary positions of the sound source. When the source or receiver is located sufficiently far from the cylinder, the new uniform asymptotic solutions reduce to well-known results for plane-wave scattering [32].

Image source solutions provide physical insights into scattering problems and are extremely easy to implement. They offer a promising way forward to simplify the complex mathematics that arises in inverse problems [8] as well as in multiple-scattering problems where the exact Green's function is not readily available [10,33–35].

This paper is organized as follows. In Section 2, the known exact solution to the problem of cylindrical wave scattering by an infinite cylinder is presented. We establish a uniform asymptotic solution for the scattered wave specific to the Rayleigh scattering regime in Section 3. In Section 4, the accuracy of the uniform asymptotic solution is numerically confirmed. Resonance scattering of sound by infinite cylinders is investigated in Section 5. In Section 6, the asymptotic solution is utilized to explore scalar and vector energy characteristics of the acoustic field in the vicinity of a target. Finally, Section 7 is a summary of our conclusions.

2. Theoretical background

In this scattering problem, an incident monochromatic cylindrical wave of frequency ω emanating from an infinite line source is scattered by inviscid fluid and elastic solid cylinders of radius a. Introduce a cylindrical coordinate system (r,θ,z) where the center of the infinite cylinder coincides with the z-axis. The source runs parallel to the axis of the cylinder and both are embedded in a homogenous fluid with sound speed c, density ρ, and acoustic wavenumber $k=\omega/c$. Since the pressure is independent of the z-coordinate, the complete description of the scattering problem can be equivalently expressed as a two-dimensional problem using the polar coordinate system consisting of the coordinates (r,θ). Cartesian coordinates are defined by $x=r\cos\theta$ and $y=r\sin\theta$ with the origin of the coordinate system coinciding with the center of the cylinder. The linear sound source is located at $r=b$, $\theta=0$.

The acoustic pressure in the incident cylindrical wave can be compactly expressed as

$$p_{in}=H_0^{(1)}(kR(b)), R(x_0)=\sqrt{(x-x_0)^2+y^2}, \tag{1}$$

where the $\exp(-i\omega t)$ time dependence is assumed and suppressed. The total acoustic pressure p is the sum of the incident (p_{in}) and scattered (p_{sc}) pressure fields, $b>a$ is the distance from the linear source to the center of the cylinder, $R(x_0)$ is the distance between points $(x_0,0)$ and (x,y), and $H_0^{(1)}(q)$ is the Hankel function of the first kind of 0^{th} order (see Fig. 1). The Hankel functions of the first kind of order n, $H_n^{(1)}(q)=J_n(q)+iY_n(q)$, are the linear combinations of the Bessel function $J_n(q)$ and the Neumann function $Y_n(q)$ [36]. The Hankel functions have the convenient large argument asymptotic form [36],

$$H_n^{(1)}(q)\sim\sqrt{2/\pi q}\exp(iq-in\pi/2-i\pi/4), |q|>>1, -\pi<\arg q<2\pi, \tag{2}$$

representing outgoing plane waves.

Outside of the cylinder, the acoustic pressure p_{sc} in the scattered wave satisfies the two-dimensional homogeneous Helmholtz equation,