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Analyzing primary resonant dynamics of functionally graded nanoplates based on a surface third-order shear deformation model

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ABSTRACT

In the context of Gurtin-Murdoch (GM) surface elasticity theory, a size-dependent third-order shear deformable plate model is developed herein in order to study the nonlinear forced vibration behavior of rectangular nanoplates with considering surface stress effect. Nanoplates are assumed to be made of functionally graded materials (FGMs) whose properties are graded in the thickness direction based on a power-law distribution. First, the constitutive relations of GM model are matricized. Then, Hamilton's principle is used to derive the governing equations. The variational differential quadrature, a numerical Galerkin, time periodic discretization, and pseudo arc-length methods are also employed for numerical solution of the geometrically nonlinear forced vibration problem. The frequency-response curves of rectangular nanoplates with different boundary conditions are investigated for different values of thickness, power-law index, surface constants and side length-to-thickness ratio. The results reveal that the surface stress has an important influence on the frequency-response curve of nanoplates at very small scales.

1. Introduction

Nanostructures including nanoshells, nanobeams, nanowires and nanoplates are extensively used in novel nanodevices such as small-scale sensors, actuators and switches. In such nano-electro-mechanical systems (NEMS), understanding the mechanical behavior of used nanostructure is of considerable importance. One way to analyze the mechanical characteristics of nanomaterials is using atomistic approaches such as molecular dynamics (MD) simulations [1–3], molecular mechanics [4–9] and density functional theory (DFT) calculations [10–12]. Since the computational cost of atomistic models are generally high (especially for nanostructures with a large number of atoms), modified continuum mechanics models are widely used as alternative models for the mechanical analyses at nanoscale. Using modified continuum models instead of classical ones is necessary as size effects play an important role in the mechanical behaviors of small-scale structures. The nonlocal [13,14], strain gradient [15–17], couple stress [18–20] and micropolar/micromorphic elasticity theories [21,22] are among the modified continuum theories which can capture small scale effects. There are several research works in the literature based on these theories [23–35].

In 1906, Gibbs [36] developed the concept of surface stress in solids. Based on Gibbs studies and those of others, atoms at or near a free surface of a solid body are under equilibrium conditions different from those for atoms in the bulk of material. Therefore, the energies of surface atoms differ from those of bulk atoms. The surface free energy is defined as the excess free energy created due to the creation of a surface in the solid, and the surface stress is defined based on the variation of the surface free energy with the surface strain [37]. For the structures at macroscale, the surface stress can be neglected because of its negligible value as compared to that of bulk stress. But, in nanostructures that have large surface-to-volume ratios, the surface stress can be no longer neglected.

Due to the important effect of surface stress on the mechanical behaviors of nanostructures, some modified continuum models have been developed in order to incorporate the surface energy influences into the

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classical elasticity theories (e.g. [38–41]). The reader is referred to two review papers on the application of surface stress-based continuum models to the problems of nanostructures [42,43]. Among the elasticity theories capable of capturing the surface stress effect, the theory developed by Gurtin and Murdoch (GM) [44,45] has attracted a lot of interest from the researchers. Based on the GM theory, the surface stress is formulated as a function of the deformation gradient, and the surface is treated as a mathematical layer with zero thickness perfectly bonded to the bulk of material without slipping. In the following, some of the investigations based upon this theory are cited.

Assadi and Farshi [46] studied the buckling behavior of circular nanoplates resting on an elastic medium based on the Kirchhoff plate theory and within the framework of GM elasticity. Hossieni-Hashemi and Nazemnezhad [47] investigated the nonlinear free vibrations of Euler-Bernoulli nanobeams made of functionally graded materials (FGMs) with considering the surface stress effect. It was revealed that, depending on the amplitude ratio, the surface stress can increase or decrease the fundamental natural frequency. Allahyari and Fadaee [48] used the GM theory to study the surface effects on the free vibrations of circular double-layer graphene sheets including geometrical defect. Ghavanloo et al. [49] developed a nonlocal model including surface effects for the modeling of breathing mode in nanowires. Rouhi et al. [50] derived the governing equations of cylindrical nanoshells based on the GM theory, and studied their free vibrations.

In the present article, the nonlinear forced vibrations of rectangular nanoplates made of FGMs are analyzed considering surface effects. The GM model is utilized to capture the surface stress effect. Moreover, Reddy's third-order shear deformation theory (TSDT) is applied to consider the shear deformation influences. The geometric nonlinearity is also taken into account according to the von Kármán hypothesis. The matricized variational expression of the problem is obtained using the variational differential quadrature (VDQ) method [51,52]. Then, the numerical-based Galerkin method [54–57], time periodic discretization [53–56] and pseudo arc-length method [57] are employed to predict the geometrically nonlinear resonance characteristics of nanoplates with the consideration of surface effects. The frequency-response curves of nanoplates with SSSS, CCCC and CSCS boundary conditions (simply-supported and clamped edges are abbreviated to S and C, respectively) are finally given for various geometrical and material properties.

2. Mathematical formulation and solution procedure

2.1. Matrix representation of GM model

Fig. 1 shows a rectangular nanoplate with length a, width b and thickness h, defined in the rectangular coordinate system $(0 \le x_1 \le a, 0 \le x_2 \le b, \quad -h/2 \le x_3 \le h/2)$. It is assumed that the nanoplate is made of a mixture of silicon (Si) and aluminum (Al), in which the material at bottom ($x_3 = -h/2$) and top ($x_3 = +h/2$) surfaces are Al-rich and Si-rich, respectively.

As indicated, the nanoplate has a bulk part and two surface layers perfectly bonded to the bulk part without slipping. For the bulk part, the constitutive equation is expressed as

$$\sigma_{ij} = \sigma_{ji} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \tag{1}$$

in which σ_{ij} and ε_{ij} denote the components of bulk stress and strain tensors, respectively; λ and μ are classical Lamé constants. Also, δ_{ij} is the Kronecker delta.

For the surface layers, based on the GM model [45], the constitutive relations are formulated as

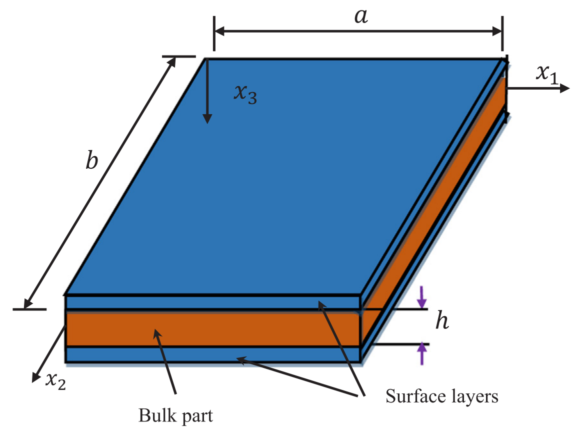

Fig. 1. Schematic of a rectangular nanoplate with bulk and surface layers.

$$\begin{aligned}\sigma^s_{\alpha\beta} &= \tau^s \delta_{\alpha\beta} + (\lambda^s + \tau^s)\varepsilon_{kk}\delta_{\alpha\beta} + 2(\mu^s - \tau^s)\varepsilon_{\alpha\beta} + \tau^s u_{\alpha,\beta} \\ \sigma^s_{3\beta} &= \tau^s u_{3,\beta}\end{aligned} \tag{2}$$

where u_i is the displacement components and $\sigma^s_{\alpha\beta}$ stands for the components of surface stress tensor. In addition, τ^s is the surface residual tension; λ^s and μ^s show surface Lamé constants.

Based on Eqs. (1) and (2) and using Voigt notation, the constitutive relations associated with to the bulk ($\mathscr{B}$) and surface layers ($\mathscr{S}$) can be written in the following matrix forms

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon} in \mathscr{B} \boldsymbol{\sigma}^s = \begin{bmatrix} \boldsymbol{\tau}^0 + \mathbf{C}^s\boldsymbol{\varepsilon}^s \\ \mathbf{0} \end{bmatrix} + \mathbf{C}^0\mathbf{D}^s\widetilde{\mathbf{U}} on \mathscr{S} \tag{3}$$

in which $\widetilde{\mathbf{U}} = [u_1 \quad u_2 \quad u_3]^{\mathrm{T}}$ is the displacement vector. By neglecting ε_{33} and σ_{33} due to the thinness of plate, one has

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_1 = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix},$$
$$\boldsymbol{\varepsilon}_2 = \begin{bmatrix} 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} \tag{4}$$

where $\mathbf{C}$ represents the bulk stiffness matrix.

Also, the Voigt form of the un-symmetric surface stress tensor $\boldsymbol{\sigma}^s$ and surface strain vector $\boldsymbol{\varepsilon}^s$ are written as

$$\boldsymbol{\sigma}^s = \begin{bmatrix} \sigma^s_{11} \\ \sigma^s_{22} \\ \frac{\sigma^s_{12} + \sigma^s_{21}}{2} \\ \sigma^s_{31} \\ \sigma^s_{32} \end{bmatrix}, \quad \boldsymbol{\varepsilon}^s = \boldsymbol{\varepsilon}_1 = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}, \quad \boldsymbol{\tau}^0 = \begin{bmatrix} \tau^s \\ \tau^s \\ 0 \end{bmatrix},$$
$$\mathbf{C}^s = \begin{bmatrix} \lambda^s+2\mu^s - \tau^s & \lambda^s + \tau^s & 0 \\ \lambda^s + \tau^s & \lambda^s+2\mu^s - \tau^s & 0 \\ 0 & 0 & \mu^s - \tau^s \end{bmatrix},$$
$$\mathbf{C}^0 = diag\left(\begin{bmatrix} \tau^s & \tau^s & \frac{\tau^s}{2} & \tau^s & \tau^s \end{bmatrix}\right), \quad \mathbf{D}^s\widetilde{\mathbf{U}} = \begin{bmatrix} u_{1,1} \\ u_{2,2} \\ u_{1,2} + u_{2,1} \\ u_{3,1} \\ u_{3,2} \end{bmatrix} \tag{5}$$