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Fully-coupled piezoelectric assumed-strain least-squares nonlinear shell

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ABSTRACT

Relevance of finite strain shell piezoelectric analysis is significant due to the general use of polyvinylidene fluoride (PVDF). A finite-strain geometrically exact shell model for the analysis of piezoelectric laminated structures is introduced. An assumed-strain formulation is employed, with least-squares fitting of *contravariant* linear *stress* fields. This allows the condensation of internal degrees-of-freedom corresponding to the assumed strains. The resulting piezoelectric shell has 8 degrees-of-freedom in each node, with 3 position/displacement degrees-of-freedom, 3 rotation parameters and the upper and lower electrostatic potential at the nodes. This contrasts with available formulations where only one electric degree-of-freedom is considered. A total of 32 degrees-of-freedom in each 4-node element are used. In term of implementation, we use a generalized strain and generalized stress formulation to reproduce the conventional finite element organization. Six examples are presented, with transversely isotropic and orthotropic cases, including finite strains and asymmetric plies. Results show a remarkably good agreement with the sources and we achieve higher values of actuation.

1. Introduction

Thin smart structures (e.g. piezoelectric), are increasingly applied in mechanical and aerospace engineering for shape control and damage detection. The great importance of this effect emerged with Paul Langevin sonar invention [20]. A complete description of applications of piezoelectricity since the early 1920's has been accomplished in [19]. In summary, applications in precise frequency control, filtering, passive signal processing, ultrasounds, actuators and motors, sensors and ignition systems (knocking sensor control) make extensive use of piezoelectric devices. Due to their fabrication and efficiency advantages, thin structures (see, e.g. [4]) and particularly shell structures (see recent advances on shells by Nguyen et al. [28]) are particularly important when dealing with smart materials. In addition, dimensional reduction can be used with great advantages in the computational cost. Robust design and simulation tools for the coupled electromechanical equations is a necessity. Specifically, for PVDF [14], strains can be significant and a full nonlinear formulation is required. A linear solution was obtained by Moleiro et al. [26] after work of Garção et al. [18]. In terms of actuation, the sources [2,17,29] are important and motivate the application with coupled finite elements. For small strains, finite element solutions are widespread, cf. [23,26,36]. For plates, shells and 3D continua applications at nanoscale, see [5–7]. However, a limited number of works exist for finite strain piezoelectric shell analysis. For beams, the analysis of Mukherjee and Chaudhuri [27] is an important reference for PVDF analysis. In essence, for finite strain shells, five relevant papers are available in this topic [21,31,37,40,41], all using a single electric degree-of-freedom per node (the difference between top and lower electrostatic potential). We extend a recently developed formulation [10], adding rotational degrees-of-freedom, coupling with the electric field and explicit zero poling stress condition. In addition, two electrostatic potential degrees-of-freedom per node are adopted, corresponding to the upper and lower shell faces. The reasons for this decision are twofold:

- Symmetry. The use of electrostatic potential at the upper and lower shell faces agrees with the kinematic shell description.
- Completeness in terms of behavior. When only one electrostatic potential, only bending piezoelectric effects can be represented. With two independent degrees-of-freedom, in-plane effects are also included in the model.

This work is organized as follows: in Section 2 governing equations are explored (equilibrium, electrostatics, piezoelectricity and global problem statement). Section 3 introduces the specific kinematics with a

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local frame and the constitutive updating. Section 4 shows our least-squares assumed strain formulation and linearization, where all strain components are fitted to the assumed-strain field. In Section 5, we present a new piezoelectric updating technique for finite strains ensuring geometrically exact results. Section 6 presents four verification tests and Section 7 presents two benchmark tests, showing excellent agreement with published data. Finally, Section 8 presents the conclusions.

2. Governing equations

2.1. Equilibrium

Using conventional notation in solid mechanics, we write the equilibrium equations as [30]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \tag{1}$$

with the Cauchy tensor components being σ_{ij} ($i, j = 1, 2, 3$) whereas $\mathbf{b_i}$ are the components of the body forces. In Eq. (1) i is the direction index and j is the facet index. Moreover, in the previous expression, x_j are the coordinates of a given point under consideration in the Lagrangian description. In addition, the following natural and essential boundary conditions hold on each part of the boundary $\Gamma = \Gamma^t \cup \Gamma^u$ where Γ^t is the mechanical natural boundary and Γ^u is the mechanical essential boundary:

$$\bar{\boldsymbol{t}} = \boldsymbol{\sigma}\cdot\boldsymbol{v} \quad \text{on} \quad \Gamma^t \tag{2}$$

$$\bar{\boldsymbol{u}} = \boldsymbol{u} \quad \text{on} \quad \Gamma^u \tag{3}$$

where $\bar{\boldsymbol{t}}$ is the known stress vector on Γ^t where $\boldsymbol{v}$ is the outer normal and $\bar{\boldsymbol{u}}$ is the prescribed displacement field $\boldsymbol{u}$ on Γ^u. As customary, conditions expressed in Eqs. (1) in the domain and (2)–(3) at the boundaries, are satisfied for a time parameter $t \in [0, T]$ with T being the total time of analysis and for a point with position $\boldsymbol{x} \in \Omega$ belonging to the deformed configuration at the time of analysis. Equilibrium configuration corresponds to the domain Ω. In tensor notation, Eq. (1) yields:

$$\nabla\cdot\boldsymbol{\sigma}^T + \boldsymbol{b} = \boldsymbol{0} \tag{4}$$

with $\nabla = \frac{\partial}{\partial \boldsymbol{x}}$ being the spatial gradient operator. After multiplication by the velocity field $\dot{\boldsymbol{u}}$, integration in the deformed configuration Ω_a and application of integration by parts component-wise, we obtain the following power form using the equilibrium configuration Ω_a ($\dot{W}_{\text{int}}$ is the internal power and $\dot{W}_{\text{ext}}$ is the external power):

$$\underbrace{\int_{\Omega_a} \boldsymbol{\sigma}\colon \boldsymbol{l}\,\mathrm{d}\Omega}_{\dot{W}_{\text{int}}} = \underbrace{\int_{\Omega_a} \boldsymbol{b}\cdot\dot{\boldsymbol{u}}\,\mathrm{d}\Omega + \int_{\Gamma^t} \bar{\boldsymbol{t}}\cdot\dot{\boldsymbol{u}}\,\mathrm{d}\Gamma}_{\dot{W}_{\text{ext}}} \tag{5}$$

where $\boldsymbol{l}$, the velocity gradient reads: $\boldsymbol{l} = \frac{\partial \dot{\boldsymbol{x}}}{\partial \boldsymbol{x}} = \frac{\partial \dot{\boldsymbol{u}}}{\partial \boldsymbol{x}}$. For algorithmic reasons, Eq. (5) is now written in a reference frame b (and configuration Ω_b) which is obtained by transformation (see, e.g. [11])

$$\int_{\Omega_b} \boldsymbol{S}_{ab}\colon \dot{\boldsymbol{e}}_{ab}\ \mathrm{d}\Omega_b = \dot{W}_{\text{ext}} \tag{6}$$

with the Cauchy stress being given by $\boldsymbol{\sigma} = \boldsymbol{S}_{aa} = \frac{1}{J_{ab}}\boldsymbol{F}_{ab}\boldsymbol{S}_{ab}\boldsymbol{F}_{ab}^T$. We introduce Ω_a as the current configuration and $\boldsymbol{F}_{ab}$ is the relative deformation gradient between configurations Ω_a and Ω_b. J_{ab} is the relative Jacobian determinant $J_{ab} = \det \boldsymbol{F}_{ab}$. In (6), $\boldsymbol{e}_{ab}$ is the relative Green-Lagrange tensor, obtained from the relative deformation gradient $\boldsymbol{F}_{ab}$ as:

$$\boldsymbol{e}_{ab} = \frac{1}{2}(\boldsymbol{F}_{ab}^T\boldsymbol{F}_{ab} - \boldsymbol{I}) \tag{7}$$

We now have to relate $\dot{\boldsymbol{e}}_{ab}$ with $\boldsymbol{l}$:

$$\underbrace{\boldsymbol{R}_{ab}^T\dot{\boldsymbol{e}}_{ab}\boldsymbol{R}_{ab}}_{\dot{\boldsymbol{e}}_{aa}} \cong \underbrace{\frac{1}{2}(\boldsymbol{F}_{0a}^T\dot{\boldsymbol{F}}_{a0}^T + \dot{\boldsymbol{F}}_{a0}\boldsymbol{F}_{0a})}_{\boldsymbol{l}} \tag{8}$$

2.2. Electrostatics

Using the electric field $\boldsymbol{E}$ as the quotient between Coulomb's force and an arbitrary test charge, charge conservation of electrostatics is expressed by Gauss's law:

$$\varepsilon_0\,\mathrm{div}\,\boldsymbol{E} = \rho \tag{9}$$

where ρ is the charge density and $\varepsilon_0 \cong 8.854 \times 10^{-12}\ \mathrm{Fm}^{-1}$ is the vacuum permittivity. In a dielectric, charge density is a sum of bound and free densities: $\rho = \rho_b + \rho_f$. Introducing the electrostatic potential ϕ, such that $\boldsymbol{E} = -\nabla\phi$, we obtain:

$$\varepsilon_0\nabla^2\phi = -\rho \tag{10}$$

For an isotropic medium, the dielectric displacement vector is defined as

$$\boldsymbol{D} = \mathscr{E}\boldsymbol{E} + \boldsymbol{P} = -\mathscr{E}\nabla\phi + \boldsymbol{P} \tag{11}$$

where $\boldsymbol{P}$ is the polarization density (to be specified later as a function of strain). In (11), we introduced the permittivity matrix $\mathscr{E}$ as:

$$\mathscr{E} = \begin{bmatrix} \varepsilon_0 & 0 & 0 \\ 0 & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_0 \end{bmatrix} \tag{12}$$

We use the charge conservation of electrostatics in the form of Gauss law to obtain a relation for $\boldsymbol{D}$:

$$\mathrm{div}\,\boldsymbol{D} - \underbrace{(\rho - \rho_b)}_{\rho_f} = 0 \tag{13}$$

where ρ_b is the bound charge density, so that $\mathrm{div}\,\boldsymbol{P} = -\rho_b$. Boundary conditions for Eq. (13) are specified on $\Gamma = \Gamma^D \cup \Gamma^\phi$ where Γ^D is the electric natural boundary and Γ^ϕ is the electric essential boundary:

$$\overline{\sigma} = \boldsymbol{D}\cdot\boldsymbol{v} \quad \text{on} \quad \Gamma^D \tag{14}$$

$$\overline{\phi} = \phi \quad \text{on} \quad \Gamma^\phi \tag{15}$$

where $\overline{\sigma}$ is the known electric displacement vector on Γ^D where $\boldsymbol{v}$ is the outer normal and $\overline{\phi}$ is the prescribed electrostatic potential on Γ^ϕ. Power density conjugacy between ρ_f and $\dot{\phi}$ results in the following weak form for (13), written for configuration Ω_b:

$$\int_{\Omega_b} \dot{\phi}\,\mathrm{div}\,\boldsymbol{D}\mathrm{d}\Omega_b = \int_{\Omega_b} \dot{\phi}\rho_f\,\mathrm{d}\Omega_b \tag{16}$$

Application of Green's first identity to (16) results in:

$$\int_{\Omega_b} \nabla\dot{\phi}\cdot\boldsymbol{D}\mathrm{d}\Omega_b = \int_{\Gamma_D} \dot{\phi}\overline{\sigma}\mathrm{d}\Gamma_D - \int_{\Omega_b} \dot{\phi}\rho_f\,\mathrm{d}\Omega_b \tag{17}$$

Introducing the configurations in $\boldsymbol{E}$ as $\boldsymbol{E}_a$ to identify the equilibrium (Ω_a) position, we have:

$$\boldsymbol{E}_a = -\nabla_b\phi_a \tag{18}$$

with $\nabla_b\phi = \frac{\partial\phi}{\partial \boldsymbol{x}_b}$ where $\boldsymbol{x}_b \in \Omega_b$ are the coordinates for reference configuration Ω_b. With these definitions, (17) can be rewritten as:

$$\int_{\Omega_b} \nabla_b\dot{\phi}_a\cdot\boldsymbol{D}_a\mathrm{d}\Omega_b = \int_{\Gamma_D} \dot{\phi}_a\overline{\sigma}_a\mathrm{d}\Gamma_D - \int_{\Omega_b} \dot{\phi}_a\rho_f\,\mathrm{d}\Omega_b \tag{19}$$

2.3. Piezoelectricity

Resorting to the mixed-set version of piezoelectricity (see, e.g. [19]) and by using our previous framework, we introduce the constitutive part of the stress as a sum of mechanical and electrical terms:

$$\check{\boldsymbol{S}}_{a0}(\boldsymbol{e}_{a0}, \boldsymbol{E}_{a0}) = \widehat{\boldsymbol{S}}_a(\boldsymbol{e}_{a0}) - \widetilde{\boldsymbol{S}}_a(\boldsymbol{E}_a) \tag{20}$$

The dielectric displacement vector $\boldsymbol{D}$ is obtained from a similar relation:

$$\check{\boldsymbol{D}}_a(\boldsymbol{e}_{a0}, \boldsymbol{E}_{a0}) = \widehat{\boldsymbol{D}}_a(\boldsymbol{e}_{a0}) + \widetilde{\boldsymbol{D}}_a(\boldsymbol{E}_a) \tag{21}$$