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Shaping of dished heads of the cylindrical pressure vessel for diminishing of the edge effect

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ABSTRACT

The paper is devoted to diminishing of the edge effect in three nonstandard dished heads of a cylindrical pressure vessel subjected to internal uniform pressure. The problem of the edge effect diminishing in the joint of the dished head with the cylindrical shell is analytically and numerically studied. The meridians of the analysed dished heads as the shells of revolution are plane curves in the Cassini oval, Booth lemniscate and clothoid forms. Geometrical relationships of the middle surfaces of the dished heads are formulated. The stress state of these dished heads are analytically and numerically studied using finite element method in Ansys system. The results of the studies are compared and presented in Tables and Figures.

1. Introduction

Stationary or mobile pressure vessels usually consist of the cylinder and two dished heads. Such structures are loaded with internal, uniform pressure. Standard shapes of the dished head i.e.: hemispherical, torispherical and ellipsoidal are subjected to the stress concentration occurring in the area of joint of the cylinder and dished head. The minimization of the stress concentration is a significant problem, particularly in designing of pressure vessels. Stress concentration phenomenon, caused by the bending of the structure in the meridional plane is called the edge effect. Reason of such structural behaviour is sudden change in the meridional curvature in a dished head.

Ventsel and Krauthammer [1] among others, described, the basis of the theory of thin-walled shells, membrane and bending states with special attention paid to the edge effect. There are many papers devoted to the problem of the edge effect minimization. Magnucki and Lewiński [2] formulated and theoretically analysed the problem of bending and shear stresses in the joint of the pressure vessels by introducing some special function describing shape of the meridian. Zingoni [3] explained the theories of membrane and bending behaviour of elastic shells, and applied those theories to numerous practical engineering cases. Magnucki et al. [4] described the minimization of stress concentration factor in cylindrical pressure vessels with ellipsoidal heads. Ortega and Robles [5] investigated a methodology of finding optimal forms of shells of revolution, which enables obtaining approximately bending-free geometries. Banichuk [6] presented shape and thickness optimization of the shell of revolution. Kisioglu et al. [7] considered strength and buckling of propane cylinder end-closures using experimental and numerical approaches. Błachut and Magnucki [8] presented a review work concerning optimization in terms of structural stability and strength of pressure vessels. Zingoni [9] presented simplification of determining influence coefficients in force method referring to various shells in non-membrane stress state. Lewiński and Magnucki [10] proposed cosinusoidal-spherical dished head shape of a cylindrical pressure vessel which highly reduced the edge effect. Kruzelecki and Proszowski [11] presented the shape optimization of dished head represented by convex Bézier polynomial. Zingoni investigated discontinuity effect in the shells junctions considering multi-segmented spherical shells [12], sludge digesters [13,14] and conical shells [15]. Pietraszkiewicz and Konopińska [16] delivered a wide review of multiple types of joints in shell structures in the aspects of stress distribution. Magnucki et al. [17] focused on elimination of the edge effect in pressure vessels. Authors proposed a meridian of dished head in form of a polynomial of the fifth degree and a circular arc. Such shape ensured significantly lower effect of the bending. Zingoni [18] reviewed recent research on the strength, stability and vibration behaviour of liquid-containment shell structures.

Due to thin-walled nature of pressure vessels, structural stability has great meaning in their analysis. Błachut [19] presented buckling of composite shallow spherical caps loaded with external pressure. Błachut [20] reviewed buckling behaviour of multiple shell structures including domed ends. Błachut [21] compared sensitivity of buckling

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load to initial shape imperfection of externally pressurised steel domes. Jasion and Magnucki described the buckling problems of Cassini ovaloidal [22] and clothoidal-spherical [23] shells under external pressure. Błachut [24] investigated buckling, collapse and first ply failure of spheroidal shells under external pressure. Zhang et al. [25] analysed buckling and post-buckling behaviour of egg-shaped shells. Zhang et al. [26] examined the effect of shape on elastic buckling of Cassini oval shells under uniform external pressure.

It is assumed within paper, the desired shape of the dished head must meet two particular conditions. Maximum stress in a cylindrical pressure vessel shall not be higher than in a cylindrical shell in the membrane stress state. The above condition implies elimination of the edge effect in the area of joint. The second condition is to achieve membrane stress state in the whole structure, maintaining possibly lowest value of the relative depth of dished head. Reasoning behind it lies strictly within practical importance i.e. applications and manufacturing process.

The subject of the analytical and numerical studies of the paper includes the following three nonstandard Cassini ovaloidal, Booth ovaloidal and clothoidal dished heads. Exemplary shapes of these curves are shown in Fig. 1. Their mathematical description is presented later in the paper.

The meridional curves are formulated in such a manner that allows to modify relative depth of the dished head, therefore their curvatures and stress distribution change. Stress concentration factor is being analysed in a function of the relative depth for the selected curves. The problem is investigated analytically as well as with the use of FE method.

2. Analytical study

2.1. Membrane stress state

Meridional and circumferential force intensity: N_1, N_2 in the shell structure loaded with internal uniform pressure are following:

$$N_1 = \frac{1}{2} p R_2, \tag{1}$$

$$N_2 = \frac{1}{2} p R_2 \left(2 - \frac{R_2}{R_1}\right), \tag{2}$$

where: p – internal uniform pressure, R_1, R_2- meridional and circumferential radius correspondingly

Stress resultants in the principal directions:

$$\sigma_1 = \frac{N_1}{t_s}, \quad \sigma_2 = \frac{N_2}{t_s}, \tag{3}$$

where t_s is the thickness of a shell.

Equivalent von Mises stress

$$\sigma_{eq} = \sqrt{\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2}. \tag{4}$$

Applying above for the cylindrical shell for which $R_1 \to \infty$, $R_2 = R_0$ one obtains

$$\sigma_{eq\ 0} = \frac{\sqrt{3}}{2} \frac{p}{t_s} R_0, \tag{5}$$

where R_0 is radius of cylindrical shell.

To compare equivalent stress in both parts of a vessel, relative equivalent stress is introduced

$$\tilde{\sigma} = \frac{\sigma_{eq\ h}}{\sigma_{eq\ 0}}, \tag{6}$$

where: $\sigma_{eq\ h}$ – equivalent von Mises stress in a dished head.

2.2. General geometry of dished head

Any dished head as a part of cylindrical pressure vessel is a surface of revolution with positive Gaussian curvature. Standard dished heads are ellipsoidal, torispherical and hemispherical. The geometrical relation for any surface of revolution is

$$\frac{d}{d\theta}(R_2 sin\theta) = R_1 cos\theta, \tag{7}$$

where angular coordinate $\theta_0 \le \theta \le \theta_1$.

Geometry of a cylindrical pressure vessel is presented in Fig. 2.

As it appears from Fig. 2 and Eq. (7) geometry of a dished head can be defined by the principal radii: R_1, R_2 in a function of the angular coordinate $0 \le \theta \le \pi/2$. Assuming that dished head is represented by a plane curve (meridian) $r(x)$ described in Cartesian coordinate system, meridional principal radius can be resolved from equation

$$R_1 = -\frac{\left[1+\left(\frac{dr}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2r}{dx^2}}. \tag{8}$$

$$R_2 = r\sqrt{1+\left(\frac{dr}{dx}\right)^2}. \tag{9}$$

Circumferential radius is defined in following manner

Length of an arbitrary meridian in Cartesian coordinate system

$$s = \int_0^h \sqrt{1+\left(\frac{dr}{dx}\right)^2}\, dx \tag{10}$$

To maintain consistency of the vessel geometry in the joint of the cylindrical shell and a dished head, following conditions must be fulfilled

$$\frac{dr}{dx}|_0 = 0, \quad r(0) = R_0. \tag{11}$$

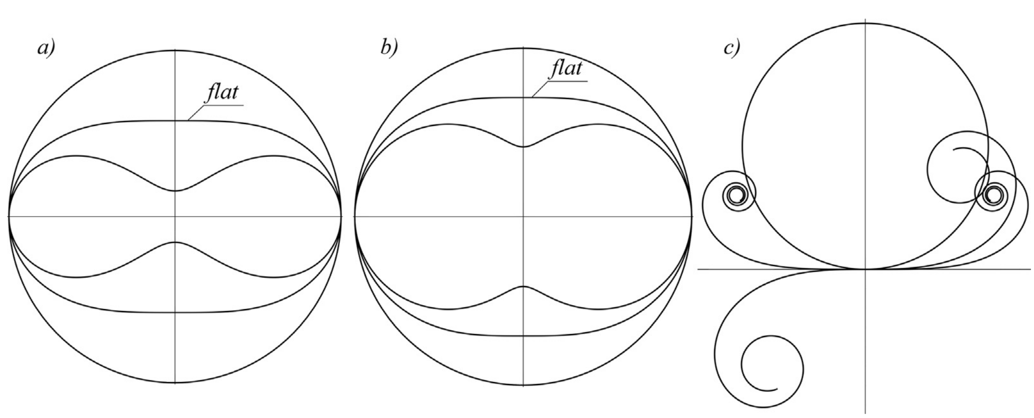

Fig. 1. Exemplary shapes of: a) Cassini ovals, b) Booth ovals, c) generalized clothoids.