Thin-Walled Structures 131 (2018) 755–768

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: A parametric analysis

Fiammetta Venuti*, Luca Bruno

Politecnico di Torino, Department of Architecture and Design, Viale Mattioli 39, I-10125, Torino, Italy

ARTICLE INFO

Keywords:
Hybrid single-layer gridshell
Instability
Free-edge
Boundary stiffness
Numerical experiments

ABSTRACT

Gridshells are form-resistant structures, which are suitable for covering large spans, especially when lightness and transparency are respectively relevant architectural and functional requirements. The majority of built gridshells are characterised by one ore more free-edges, which derive from trimming the gridshell reference surface in order to provide building access or to integrate the gridshell within existing structures. Up to now, only few scientific systematic studies have been devoted to the effects of elastic boundary structures on the stability of gridshells. This study aims at filling some gaps about this issue. To do so, an ideal free-edge bending-inactive hybrid single-layer gridshell is analysed. The gridshell sensitivity to the flexural stiffness of the boundary arch and to the shear stiffness of the gridshell are investigated through an extensive parametric analysis, which was performed by means of numerical experiments. Results are first discussed in terms of the well-established load factor and buckling shape. Then a complementary mechanical reading is provided by introducing ad-hoc conceived local metrics of the in-plane and out-of-plane deformations at collapse. Three different mechanical regimes at collapse are outlined. In conclusion, a range-finding chart within the design parameter space is proposed to orient the structural analyst in the choice of the preferred regime.

1. Introduction

Gridshells are form-resistant structures, which are designed to ideally bear the loads by means of in-plane internal forces. Their geometry is generally defined, at least in steel gridshells, by approximating a reference continuous surface through a discrete pattern of line-like structural members, which are mainly subjected to axial forces.

Gridshells find their natural application in large-span buildings, such as stadia, courtyards and expo pavilions, where transparency and lightness might be relevant program requirements. Not surprisingly, the first known pioneering application of a doubly-curved gridshell, which dates back to the late 19th century and was designed by engineer Vladimir Shukov, refers to the roof of a large-span Plate Rolling Workshop in Russia [1]. The geodesic dome of the Zeiss-Planetarium in Germany, designed by Walther Bauersfeld and completed in 1926, is another example of this kind. Throughout the second half of the 20th century, the milestone achievements of Buckminster Fuller [2], Frei Otto [3] and Schlaich Bergermann und Partner [4,5] helped defining the current technology and strategies for gridshell design: on the one hand, the work of Frei Otto gave birth to what is currently known as a bending-active, or post-formed, timber gridshell [6,7]; on the other hand, Jörg Schlaich and Hans Schober focused on bending-inactive, or pre-formed, steel/glass gridshells, most of which were derived from surfaces of translation and were based on the use of quad patterns [8]. Since these pioneering structures, the gridshell structural concept has been widely applied to a variety of buildings all over the world, both in traditional materials [5,9] and innovative ones [10–12].

Gridshells are optimised and highly efficient structures, but this efficiency makes them highly prone to buckling phenomena, which can lead to catastrophic collapse. Since the collapse of the Bucharest Exhibition Hall dome in 1963, a lot of research has been devoted to buckling and post-buckling behaviour of reticulated shells [13–20]. The first approach to the problem was based on the continuum analogy, that is, the behaviour of the gridshell is compared to the one of a shell, characterised by the same geometry of the gridshell and by an equivalent thickness [21–24]. The aim of this approach was to extend to gridshells the analytical expressions of the buckling load that were available for continuous shells. This method, even though it can be useful in a preliminary design phase, presents some drawbacks [13]: analytical solutions are available only for a class of continuous shells, characterised by specific shape and boundary conditions; the continuum analogy does not allow to account for some types of buckling that are peculiar of gridshell structures, such as nodal buckling and member buckling; the influence of joint deformation cannot be taken

* Corresponding author.
E-mail address: fiammetta.venuti@polito.it (F. Venuti).

https://doi.org/10.1016/j.tws.2018.07.019
Received 22 December 2017; Received in revised form 17 April 2018; Accepted 10 July 2018
Available online 22 August 2018

Nomenclature

GMNA	Geometrically and Materially Nonlinear Analysis
LBA	Linear Buckling Analysis
LF	Load Factor
CG	Complete Gridshell
PG	Partial Gridshell
E	modulus of elasticity
L	dome span length
f	dome rise length
l	characteristic length of quadrilateral face
p	subscript of the generic structural node
P	number of structural nodes
x	horizontal space coordinate
y	horizontal space coordinate
z	vertical space coordinate
K	discrete gaussian curvature
$\mathbf{K}_e$	elastic stiffness matrix
$\mathbf{K}_g$	geometric stiffness matrix
I_b	moment of inertia of the boundary structure
I_e	moment of inertia of the grid elements
I_r	ratio of the boundary to grid moments of inertia
S_r	non-dimensional ratio between design parameters
Q	nodal resultant load
q	uniform load
s	uniform live (snow) load
g	dead load
A	quadrilateral surface
A_e	cross section area of grid elements
A_c	cross section area of diagonal cables
f_y	yield strength
ΔK	out-of-plane nodal deformation
γ_{xy}, γ_{yx}	components of the in-plane nodal deformation
Γ	in-plane nodal deformation
ν	nodal direction
μ	load multiplier
σ_0	cable initial prestressing
σ	stress field
ϕ	buckling mode shape
φ	generic buckling shape
λ	eigenvalue
δ	nodal displacement
δ_u	ultimate nodal displacement
κ	cotangent of the nodal angle

into account. For all these reasons, the buckling behaviour of gridshells is usually studied by means of non-linear numerical analyses of Finite Element Models of the whole structure.

The main factors, which influence the buckling behaviour of gridshells and that have been studied in the literature, are the following: [13,25]: i. *the Gaussian curvature* of the gridshell underlying surface (e.g. [13,16,19,26]); ii. *the grid topology and spacing* (e.g. [20,24,25,27]); iii. *the geometrical and mechanical imperfections* (e.g. [26,28–32]); iv. *the joint stiffness* (e.g. [16,29,33–35]); v. *the boundary conditions*.

While the effects of the first four factors have been widely studied in the recent past, the last factor has been less explored. Stability studies on gridshell domes are usually carried out by referring to a horizontal spring-plane and rigid supports [15,18,20,24,26,27,29,36–39]. The effects of different rigid supports (pinned, roller or fixed) have been comparatively studied in a few papers [13,19,40,41]. Analogously, the gridshell barrel vaults studied in the literature are usually delimited by a horizontal spring-plan and gable vertical plans, which are orthogonal to their axes [19,26,39] and rigidly constrained at the spring-lines. To the authors' knowledge, the effects of different rigid supports (fixed or pinned) along the spring-lines of a gridshell barrel vault have been discussed in [19,42] only.

Even fewer scientific systematic studies have been devoted up to now to the effects of elastic boundary structures along the delimitation edges on the stability of gridshells, as recently highlighted in [42]: "This kind of elastic boundary has not been extensively investigated, and studies are needed in each design to know how and if the supports improve the buckling resistance.". This is even more surprising knowing that a horizontal spring line and/or infinitely stiff perfect constraints only seldom occur in built gridshells. Usually and more and more frequently, gridshells are trimmed by vertical or inclined planes (e.g. the Shukov gridshell in Vyksa [1]), or by curved surfaces (e.g. the Hippo House at the Berlin Zoo [43]). As a consequence, they are constrained by elastic boundary structures along the delimitation edges. Master builders were certainly aware of the influence of elastic boundaries, as demonstrated by the above cited built structures. However, to the authors' best knowledge, systematic studies applied to double curvature single-layer gridshells are not available, and only a few number of studies analysed the effects of the stiffness of gable boundary structures in barrel vault gridshells. Bulenda and Knippers [26] compared the barrel vault stability with no stiffening boundary arches with the same structure stiffened by a boundary arch having 40 times the stiffness of a IPE 360 profile. As expected, the differences in terms of mechanical behaviour are striking: the unstiffened vault buckles analogously to a plane arch, while a spatial behaviour at collapse takes place in the stiffened vault, even if the stiffening arch still shows not negligible ultimate displacements. Cai et al. [19] stiffen the vault with pre-tensioned spoked wheels transverse diaphragms, analogously to the stiffening system adopted for the roof of the Museum of Hamburg History courtyard [4]. In this case, the arch acts as a perfectly rigid support in its vertical plane.

This issue seems to be disregarded also in the case of continuous shells. A number of remarkable free-edge concrete shells were conceived by master builders such as Heinz Isler [44] and Felix Candela [45]. In their works several solutions were put in place at the shell free boundaries, ranging from unstiffened edge, to creases introducing form stiffness, to edge beams providing inertial stiffness. However, once again "the stability of shells with free edges is a rather unexplored field" [46]. In fact, even though single curvature cylindrical shells with flexible bottom have been widely systematically studied over the past fifty years, and their buckling behaviour is well known ([47–51]), analytical solutions for double curvature shells with elastic boundaries are not available in literature.

In summary, a scatter exists between the design practice and the scientific literature in this field. In this framework, the paper focuses on the delimitation of gridshells, and aims to fill in some of the gaps, which are currently evident in the topic literature review. In terms of paper outline, Section 2 defines, classifies and reviews some relevant examples of free-edge gridshells, both from the geometrical and mechanical points of view. The term Partial Gridshells (PG) will be introduced here to denote free-edge gridshells as opposed to Complete Gridshells (CG). A new parametric study is described in Section 3, in terms of geometrical setup, structural setup and design parameter space. Section 4 briefly recalls gridshell structural modelling and computational approaches. The observables selected for post processing are described in Section 5. Results and findings of the parametric analyses are described and discussed in Section 6, in parallel with a resulting range-finding chart, developed in the design parameter space. Section 7 summarises the conclusions and future works.