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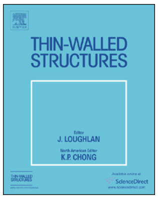

Full length article

A refined layerwise finite element modeling of delaminated composite laminates with piezoelectric layers

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ABSTRACT

A finite element model for static response and free vibration analysis of delaminated composite laminates with active layers is developed based on a coupled layerwise laminate theory. The proposed model assumes full layerwise variations for both mechanical displacements and the electric field, and more importantly, both strong and weak discontinuous functions are introduced into the displacement fields to model the displacement discontinuity induced by the delamination and strain discontinuity induced by the interface between the layers, respectively. This refined layerwise mechanics can give a more accurate description of displacement discontinuity. The formulation naturally includes the coupling and interactions between the composite laminates and piezoelectric layers. Various examples are investigated by the developed model and the results are compared with previously obtained alternative solutions or experimental results to verify the justification and accuracy of the present formulation. Finally, the effect of delamination on static response and vibration mode are studied in detail to provide beneficial information for the promising damage identification techniques.

1. Introduction

Composite laminated structures with piezoelectric layers have received considerable attention in recent years due to their coupled mechanical and electrical properties. Structures with embedded or surface mounted piezoelectric sensors and actuators have been widely used in engineering applications. Recently, vibration and shape control using piezoelectric actuators and sensors is an area experiencing significant technical activity. Particular emphasis is placed on the active structural monitoring techniques for composite structures. It is widely recognized that the composite laminated structures are very prone to interlaminar defects arising between two anisotropic layers which results in the high interlaminar stresses caused by geometric discontinuities or mismatch in the material properties. One candidate type of the defect is the delamination crack, which is the most dangerous failure pattern for composite materials particularly if induced by fatigue and low velocity impact. Among the various open issues in this area, researchers focus primarily on the development of analytical and numerical models qualified for capturing the effect of delaminations on the structural electromechanical response associated with the piezoelectric sensors and actuators, as the sensitivity of electromechanical response on damage characteristics may be investigated thoroughly by such models, and hence based on such models, damage detection and localization techniques of the intelligent composite system can be developed. To this end, this paper presents a full layerwise finite element formulation for analyzing the static and modal response of delaminated composite beams and plates with active layers, in which both strong and weak discontinuous function are taken into the displacement fields to model the displacement discontinuity induced by the delamination and strain discontinuity induced by the interface between the layers, respectively.

The pioneering attempts in this field have been focused on the effects of delaminations on the modal response of composite beams [1–6] and plates [7,8] using classical beam and plate theories. Thereafter, Luo and Hanagud [9] derived an analytical theory using the Timoshenko beam theory, which is capable of predicting the static, modal and dynamic response of delaminated composite beams. The model introduced piecewise linear springs between the delaminated sublayers to simulate the “open” and “closed” behavior. Using a higher-order displacement theory, Thornburgh and Chattopadhyay [10] presented a model to predict the behavior of composites laminates with delamination and transverse matrix cracking, which enables the independent description of displacement fields above and below the delamination. Hu et al. [11] proposed a FEM model based on a simple higher-order plate theory to analyze the vibration response of delaminated

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composite plates. Chrysochoidis and Saravanos [12] investigated both experimentally and analytically the effect of delamination on the vibration response of composite beams with surface mounted piezoelectric sensors and actuators. The sufficiency and advantages of the proposed piezoelectric actuator-sensor pairs in self-monitoring were demonstrated by comparing to other traditional sensors. Kim et al. [13] proposed an improved layerwise laminate theory of delaminated smart composite laminates by imposing a Fermi-Dirac distribution function, which has the capability of computing interlaminar shear stresses. Tenchev and Falzon [14] presented a new interface elements based on a pseudo-transient formulation, which was demonstrated through the modeling of a double cantilever beam undergoing Mode I delamination. Cappello and Tumino [15] performed a study on the buckling and postbuckling behavior of unidirectional and cross-ply composite laminated plates with multiple delaminations using a linear buckling model. Using the Timoshenko beam theory and both the ‘free mode’ and ‘constrained mode’ assumptions in delamination vibration, Liu and Shu [18] presented an analytical solution to study the free vibrations of rotating Timoshenko beams with multiple delaminations. Alidoost and Rezaeepazhand [19] proposed an analytical solution for analyzing the vibration and buckling instability behavior of delaminated composite beams with a single delamination using Euler-Bernoulli beam theory and Classical Lamination Theory. More recently, improvements were made to layerwise theory to analyze the static, dynamic and buckling behaviors of composite laminated plates and shells with multiple delaminations [20–23]. Experimental methods were also performed for identifying preexisting delamination cracks in composite laminates using embedded piezoelectrics [24–26] and predicting the effect of multiple delaminations on natural frequencies of composite laminates [27,28].

A substantial amount of investigations have been conducted on the modeling strategies of delamination in composite laminates. Generally, previous works mainly employed Heaviside step function into the displacements fields through the thickness direction to simulating the delamination. Barbero and Reddy [29] extended the Reddy's layerwise laminate theory to describe multiple delamination cracks between layers. The jump discontinuous conditions of displacements were implemented by introducing new degrees of freedom allowing for the sliding and opening across the faces of the delamination. Chattopadhyay and Gu developed a new higher order shear deformation theory for modeling delamination buckling and postbuckling of composite laminates [30] and cylindrical shells [31]. At the delaminated interfaces, delaminations between layers of composite plates and shells were modeled by the Heaviside step function with both lower and higher order terms of displacements taken into account. Higher order terms of displacements are identified at the beginning of the formulation by assuming that shear stresses vanish at all free surfaces including the delaminated interfaces. Kumar et al. [32] proposed a new method of modeling for partial delamination in composite beams by using a two nodded C1 type Timoshenko beam element with 4 degrees of freedom per node. Williams [33] proposed a generalized multi-length scale theory of composite laminated plates with delaminations based on a generalized two length scale displacement field assumption obtained from a superposition of global and local displacement effects. By the appropriate simplification, this displacement field can be reduced to any currently available theory, such as the variationally derived, displacements based (discrete layer, smeared, or zig-zag) plate theory. Chrysochoidis and Saravanos [34] formulated a coupled linear layerwise laminate theory and a beam FE for analysis of delaminated composite beams with piezoelectric sensors under zig-zag fields assumption for axial displacements and the electric potential, which modeled the discontinuity in displacement fields due to the delaminations as additional degrees of freedom. The static and modal response of piezoelectric composite beams with various delaminations sizes was well predicted. Cho and Kim [35] presented a higher-order zig-zag theory for the laminated composite plates with multiple delaminations by imposing top and bottom surface transverse shear stress-free conditions and interface continuity conditions of transverse shear stresses including delaminated interfaces. The influence of the number, shape, size, and locations of delaminations on the responses can be taken into account systematically by using this displacement field model. Li and Qing [36] developed a semi-analytical three-dimensional model for composite laminated shells based on the modified Hellinger–Reissner variational principle. A nonlinear spring-layer model between the exterior and interior sub-laminates is proposed to simulate the delaminated region. In most recent years, discontinuity integrated into a distinct continuous solution [37,38] was investigated through properly designed coupling schemes between differential based and integral based solutions.

A review of the literature shows that, some finite element models have been based on the equivalent single-layer laminate (ESL) theories, and the others have been based on higher-order shear deformation theories (HSDT) and partial layerwise displacement theory. Both cases are insufficient to give an accurate mechanical and electric field at ply level for delaminated piezocomposite plates. Moreover, no attempt is made to introduce weak discontinuity induced by the interface between the layers into the full layerwise displacement theory. To overcome this deficiency, this paper presents a refined full layerwise displacement field, in which both strong and weak discontinuous functions are introduced into the displacement fields to model the displacement discontinuity induced by the delamination and strain discontinuity induced by the interface between the layers, respectively. Based on the proposed coupled layerwise mechanics, a finite element formulation for delaminated composite plates with piezoelectric layers has been developed which is capable of representing more refined description of the kinematics of laminated structures so that the interlaminar stresses and electric potential can be obtained more accurately. The proposed formulation is verified with various numerical examples, and effect of embedded delamination in a piezocomposite plate is further investigated for static response and free vibration analysis.

2. Displacements and electrical potential field assumption

A generalized laminate theory called Layerwise Laminate Plate Theory (LWPT) was proposed by Reddy [39] that gives a scheme from which any of the displacement-based, 2-D laminate theories can be derived. The LWPT of Reddy provides a more realistic description of the displacement field by introducing discrete layer transverse shear effects and discrete layer transverse normal effects. The accuracy of layerwise kinematic model can be easily improved by using more linear sub-divisions through the thickness or employing quadratic variation through the thickness.

To model strain discontinuity induced by the interface between the layers, the interpolation nodes through the thickness direction are assigned at the top surface, the bottom surface, and the middle surface. For the smart composite laminated plates with delamination, the node strategy proposed in the present paper is illustrated in Fig. 1. Where h_k denotes the thickness of the k-th sub-layer, z_k represents the z-coordinate of the interface between k-th and (k-1)-th. N is the number of nodes through the thickness, which implies the N-2 number of interfaces between the layers.

In the layerwise theory of Reddy [40], the total displacement field for the lamina within a discrete layer, taking into account N_D delaminations and N-2 interfaces between the layers, is represented as