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Unified theory for curved composite thin-walled beams and its isogeometrical analysis

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ABSTRACT

This paper presents a unified theory for modelling composite thin-walled beams (TWB) of arbitrary planar axial curvature, variable cross-section and general material layup, complemented by the development of an Isogeometric Analysis (IGA) formulation for the discretization and solution of the equilibrium equations. To this end, the standard formulation of composite TWB with rectilinear axes is combined with a general framework for describing the kinematics of arbitrary three-dimensional curves based on the Frenet-Serret frame field. The theory includes explicit terms accounting for curvature gradients within the IGA stiffness matrix, allowing for the treatment of cases with highly curved geometry. Also included is an advanced shear-modification adjustment previously derived for rectilinear TWB, here reformulated for the case of curved TWB, improving the description of the in-plane shear-strain coupling and thus increasing the accuracy for cases with axial-bending-torsional structural coupling. Results from three numerical test cases indicate that this unified formulation effectively transfers all the advantages associated with rectilinear TWB to curved TWB models, yielding an accuracy comparable to more complex models while maintaining a competitive computational economy.

1. Introduction

Thin-walled composite structures have widespread applications in industries such as aeronautical (e.g. wings and fuselages), civil (e.g. bridges and pipes), space (e.g. antennas and containers), and energy (e.g. wind turbine blades), and are quickly replacing metallic counterparts in structural applications where a favourable stiffness-to-weight ratio is critical [1–7]. Some of the above examples can be modelled as thin-walled, beam-like composite structures with a varying degree of axial curvature, a feature that cannot be overlooked as even a slight amount has a significant impact on structural rigidity and multiaxial coupling. Nonetheless, theories describing the mechanics of composite thin-walled beams (TWB) with arbitrary axial curvature, variable cross-section, and general material layup are scarce, with most of them exhibiting some kind of restriction such as constant in-plane axial curvature, prismatic and/or homogeneous cross sections, or a specific arrangement of material layup [8–39].

In the context of this work, a composite TWB model refers to a thin-walled beam structure which admits variable geometry and material layup along both the cross section's contour and the beam axis, allowing to capture, via semi-analytical equations, stiffness coupling both at the cross-sectional level and at the beam structure as a whole. All this information is reduced to a single axis whose displacement field is represented by a few global degrees-of-freedom (DOF). Despite its compact size, the model allows, under a few key assumptions, for the recovery of the strain and stress fields with greater detail (i.e. strain/stress tensor at individual laminas) than standard beam models and with an accuracy comparable to more sophisticated (i.e. 3D-Shell) models, while maintaining a competitive computational economy. The modelling of TWB with rectilinear (i.e. straight) axis is already a mature field thanks to a considerable body of research and great advances made particularly during the last two decades, with main developments due to Kollar and Pluzsik [8], Librescu and Song [9], Lee et al. [10], Kim et al. [11–13] and others. Recent advances include the one by Zhang and Wang [14,15] who presented an improved treatment of the in-plane shear-strain coupling, providing a significant increase in

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accuracy for cases with axial-bending-torsional coupling. It must be highlighted that the type of composite TWB model discussed in the present work is different from standard beam models that also admit thin-walled cross-sections but offer reduced capabilities in comparison [16], and also differs from other advanced beam models, some of them accounting for the deformation of the cross-section (solid or thin-walled), but at the expense of a complex numerical pre-processing step in order to obtain the cross-section's stiffness [17–21].

The complexity of curved beam models, on the other hand, has gradually grown over the years. Approaches have evolved from early models based on a number of simultaneous restricting assumptions such as isotropic materials [22–26], the absence of coupling between different DOF [27–30], prismatic beams and/or homogeneous cross-section properties [31–33], and constant curvature [34–36], among others, towards models where some of these restraints have been increasingly relaxed. One important innovation that greatly facilitates the modelling of curved beams is the use of the Frenet-Serret frame field, which describes the beam kinematics based on a local reference frame given by the tangent, normal and bi-normal vectors along the curved axis (see e.g. Akdz et al. [28]). Lee and Thompson [37] used a Frenet-Serret formulation to solve the problem of dynamical stiffening of helical strings, a common problem in curved-beam analysis. In spite of those advances, a unified theory capable of handling arbitrary cases of curved TWB is still missing, and a literature survey conducted by the authors was able to identify only two works, [38,39], capable of handling curved TWB models as defined in this paper. The first of these references [38] introduced a full-featured TWB model which was however limited to a constant in-plane curvature while the second [39] presented a simplified TWB model (i.e. with reduced capabilities for modelling the axial-bending-torsional couplings considered in full-featured counterparts), albeit with variable curvature of the beam axis or *pole line*. The present work overcomes these limiting assumptions by providing a framework which includes a full-featured TWB model and the consideration of arbitrarily curved pole lines.

As far as the method for the solution of the structural equations is concerned, the literature survey shows that the vast majority of models are solved via some type of finite-element method (FEM), with only a few using semi-analytical methods or Isogeometrical Analysis (IGA) approaches. IGA, originally developed by Hughes, Bazilevs, and coworkers [40,41], builds the equilibrium equations directly from geometry-based basis functions (Non-Uniform Rational B-Splines, or NURBS, the main engine in most Computer-Aided Design platforms), entirely by-passing the need of a mesh-geometry (a compulsory step in standard FEM) and therefore avoiding a large portion of the time needed to setup a model; a reduction of up to 80% was estimated by one reference [40]. Other advantages associated to IGA formulations include a smaller system size, improved numerical properties resulting in faster solution times, a direct link between the model geometry and the resulting equations of motion, a smoother curvature field, and a straightforward handling of shear-locking [40–44]. Due to the advantages mentioned above, isogeometric analysis has been used to solve problems in a wide range of engineering fields such as: structural mechanics of beams, shells and plates [16,45–55], damage and fracture mechanics [56–58], contact mechanics [59–61], structural shape optimization [44,62–64], fluid dynamics, fluid-structure interactions, and electromagnetics [65–71].

Though the discussion above provides evidence for a significant evolution of advanced beam models, including those specifically formulated to address initially curved beams, in recent years the authors of the present work have been unable to identify formulations capable of handling curved TWB of arbitrary (initial) curvature of the beam axis, variable cross section, and general composite layup. The present work is intended to provide such a unified theory. To this end, a general description of arbitrary three-dimensional curves based on the Frenet-Serret frame field is combined with the standard theory of rectilinear TWB, providing a simple and elegant transformation from the equations of motion for the rectilinear case to their curved counterparts. In this formulation curvature and torsion gradients arise naturally, allowing for the treatment of cases with highly curved geometry while fully considering the associated increase in rigidity. Additionally, the in-plane shear strain formulation proposed by Zhang and Wang [14,15] for rectilinear TWB, shown in references [14,15] to significantly increase the accuracy in cases involving axial-bending-torsional coupling (a common scenario for composite materials), is reformulated for curved TWB. A further contribution of this work is the development of an isogeometrical (IGA) formulation for discretizing and solving the model equations. The resulting formulation – while having the advantage of being far more general than previously published thin-wall beam theories in terms of curvature variation and materials layup – is also extremely lean in terms of its requirement of computational resources, allowing for fast iterations and sensitivity studies. Limitations include the general restriction of analytical TWB models to undeformable sections, as well as the fact that the current work is based on a Bernoulli formulation, although a generalization to a Timoshenko-type model is relatively straightforward.

Three numerical test cases are formulated in order to discuss some of the specific contributions of the new unified theory developed in this work. The first test case shows how the curvature gradient of the beam axis or *pole line* in Euler-type models significantly modifies the displacement field. Since the corresponding terms are often omitted these results may result interesting to the TWB modelling community. As mentioned above, curvature (and torsion) terms arise naturally in the present formulation and do not require additional considerations. It is worth mentioning that these terms only arise explicitly in Euler-type beam models, whereas changes in curvature and torsion of the beam axis are implicit in Timoshenko-type formulations. A Timoshenko-type version of the current theory will be presented in follow-up work. The second test case demonstrates the effect of couplings between degrees of freedom on the shear strain in curved thin-wall beams, building on references [14] and [15] where the authors demonstrate that a radical improvement in accuracy is obtained in cases with significant axial-bending-torsional coupling, by deriving the shear strain from the constitutive equations of the corresponding laminate. The curved-beam versions of the equations of references [14,15] developed in the present work allow to study the effect of non-symmetrical or non-balanced layups in more general cases. Test case 2 specifically addresses a one-ply layup with varying fibre angle in order to demonstrate the increases in accuracy in a very clean test situation. Test case 3, finally, demonstrates how the different elements of the unified theory presented here work together nicely in a realistic case showing significant complexity.

The rest of the paper is organized follows: Section 2 first develops (in subsection 2.1) the well-known kinematic equations of rectilinear thin-wall beams, i.e. beams having a straight beam axis or *pole line* in the absence of loads. Apart from allowing the reader to recall some textbook results the description makes sure the nomenclature required in the following is well understood. A specific contribution of subsection 2.1 is the introduction of the $(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b})$ frame which naturally morphs into the Frenet-Serret traveling tripod used in differential geometry in subsection 2.2. The latter develops the curved-beam versions of the TWB kinematic equations by explicitly considering derivatives of the basis vectors $(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b})$ in addition to the usual derivatives of the displacement variables. It will become immediately clear that curvature and torsion gradient only appear explicitly in Euler-type formulations, though their effect is implicit in shear-deformable formulations. A separate section (Section 3) is dedicated to the isogeometrical formulation of curved thin-wall beams, showing how the displacement field can be determined in a compact and neat way. Section 4 has a description and discussion of the test cases mentioned above, and Section 5 provides a summary and concluding remarks. A significant portion of the equations developed in the course of this work are relegated to several appendices in order not to impact the free flow of the narrative.