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Analytical solution for tunnels not aligned with geostatic principal stress directions

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ABSTRACT

It is well-known that rock masses may present marked stress anisotropy. However, most of the tunnel analyses (numerical and analytical) assume the tunnel axis aligned with one of the principal stress directions. When this is not the case, axial shear stresses appear, which then are neglected, as it is done in all analytical solutions available for tunnel analysis. Existing solutions may consider advanced nonlinear ground behavior (i.e. elastic-brittle-plastic with e.g. Hoek and Brown failure criteria), linear-elastic ground with transversely anisotropic properties, seismic loading, groundwater and support, etc., but all consider that the axis of the tunnel aligns with one of the principal far-field stresses. This is also what is generally assumed when conducting more sophisticated, three dimensional numerical analyses. In this paper, an analytical solution to calculate the stresses and displacements induced by far-field axial shear stresses is presented. Solutions for supported and unsupported tunnels are provided. The proposed analytical solution can be combined with the classical Kirsch and Einstein-Schwartz solutions to determine the complete stress and displacement fields around the tunnel. Further, the effects of stress anisotropy are discussed.

1. Introduction

Analytical solutions have been extensively developed for tunnels. Some of them are regularly used in practice, such as the Kirsch and Einstein-Schwartz solutions (Kirsch, 1898; Einstein and Schwartz, 1979). Despite improvements on numerical modeling, analytical solutions are still used because they allow fast and robust tunnel analysis. For instance, Ledesma and Alonso (2017) obtained accurate ground deformation predictions caused by tunnels under the World Heritage Structures "Sagrada Familia Basilica" and "Casa Mila", in Barcelona, Spain, using analytical solutions. For reliability problems, which may require a large number of calculations, analytical solutions are widely used because numerical methods may be unmanageable or even unfeasible. Analytical solutions are attractive because they incorporate the most significant variables in a closed-form formulation and are benchmarks to sophisticated numerical analysis and code validation. However, the mathematical treatment of analytical solutions may be cumbersome and simplifications must be assumed. The analytical solutions for tunnels normally rely on 2D plane strain conditions and circular tunnel cross-sections.

New analytical solutions for tunnels are being developed. For example, the solutions proposed by Kirsch and Einstein Schwartz were expanded by Bobet (2003) to incorporate the effects of groundwater flow and seismic loading for lined and unlined deep tunnels in linear elastic ground. Further expansions, to include transversely anisotropic elastic ground, were carried out by Hefny and Lo (1999), Bobet (2011), Zhang and Sun (2011), Bobet and Yu (2016), Bobet (2016a, 2016b). Analytical solutions for viscoelastic ground are also available. Those solutions were proposed by Wang et al. (2013, 2015, 2017) for deep tunnels with elliptical cross-section and also for circular twin tunnels. The analytical solutions mentioned so far are applicable to deep tunnels. Analytical solutions for shallow tunnels in linear-elastic ground were presented by Bobet (2001), Park (2005), Pinto and Whittle (2014), Strack and Verruijt (2002), Verruijt and Booker (1996) and Verruijt (1997). The applicability of analytical solutions for shallow tunnels was assessed by Chou and Bobet (2002) and Pinto et al. (2014). Both papers found good agreement between field data and predictions using analytical methods.

The closed-form solutions found so far assume elasticity and, thus, are valid only if minor or no yielding is present around the opening. Including plasticity in the solutions increases the complexity of the problem, and results are currently limited to unsupported tunnels, static loading, dry ground and isotropic far-field stresses. Salesçon (1969) developed an analytical solution for a loaded hollow plate in elastic

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Nomenclature

List of variables

α	angle between tunnel axis and major horizontal principal stress
$\sigma_{xx,ff}$	far-field horizontal stress normal to the tunnel axis
$\sigma_{yy,ff}$	far-field vertical stress, assumed normal to the tunnel axis
$\sigma_{zz,ff}$	far-field stress parallel to the tunnel axis
$\tau_{zx,ff}$	far-field axial shear stress
K_{xy}	stress ratio ($\sigma_{xx,ff}/\sigma_{yy,ff}$)
w	axial displacement
u_r	radial displacement
$G = G_g$	shear modulus of the ground
G_s	shear modulus of the structure
E	Young's modulus
ν	Poisson's ratio
r_0	tunnel radius
t	support thickness
r_i	tunnel internal radius (r_0 + t)
T	thrust force of the liner
M	bending moment of the liner
V	axial shear force of the liner
ϕ	friction angle of the ground
c	cohesion of the ground
Ψ	dilatancy angle of the ground
x, y, z	coordinate system attached to the tunnel, with z-axis parallel to tunnel axis
σ_v	vertical stress
σ_h	minor principal horizontal stress
σ_H	major principal horizontal stress
r, θ, z	cylindrical coordinate system, with z-axis parallel to tunnel axis
$\sigma_{\theta\theta}$	tangential stress in cylindrical coordinates
σ_{rr}	radial stress in cylindrical coordinates
$\tau_{r\theta}$	in-plane shear stress in cylindrical coordinates
τ_{rz}	axial shear stress in cylindrical coordinates
$\tau_{\theta z}$	axial shear stress in cylindrical coordinates

perfectly plastic ground with Mohr-Coulomb failure. Such solution applies to deep tunnels. The solution considers associated and non-associated flow rules. Other solutions, for other failure criteria are currently available, for example the solution from Carranza-Torres (2004) with the Hoek and Brown failure criteria. Sharan (2003, 2005) incorporated elastic-brittle-plastic behavior with Hoek and Brown failure in an analytical solution. Massinas and Sakellariou (2009) found an analytical solution for shallow tunnels considering elastic perfectly plastic material with Coulomb failure.

In all the formulations discussed, the tunnel axis is aligned with one of the principal directions because all solutions assume plane strain conditions on any cross-section perpendicular to the axis of the tunnel. Therefore, the far-field axial shear stress that appears due to the misalignment of the tunnel with the horizontal principal stresses is neglected.

It is well-known that rock masses may have pronounced anisotropic far-field stresses (Brady and Brown, 2006; Jaeger et al., 2007; McGarr and Gay, 1978). Under these conditions, the plane strain assumption may be incorrect and may lead to erroneous conclusions (Hoek, 2008). The importance of the orientation of the underground excavation with respect to the far-field stress tensor is well-recognized in choosing the orientation of caverns and their shape. It is generally recommended to orient them parallel to the major principal stress direction, and with a shape such that stress concentrations are minimized (Goodman, 1989). However, for most applications in Civil engineering, the tunnel alignment is pre-determined and must be designed regardless of its orientation with respect to the far-field stress tensor.

McGarr and Gay (1978) determined the complete geostatic stress tensor from 77 different sites. From their compilation, it is possible to estimate the level of stress anisotropy expected in rock masses. Fig. 1 shows the scatter of the principal stress ratios compiled by McGarr and Gay (1978) (σ_1/σ_2, σ_1/σ_3, σ_2/σ_3) with depth. The average, plus or minus one standard deviation, for each principal stress ratio is: $\sigma_1/\sigma_2 = 1.45 \pm 0.40$; $\sigma_1/\sigma_3 = 2.42 \pm 1.14$; and $\sigma_2/\sigma_3 = 1.66 \pm 0.5$. Those ratios may be even higher for shallow depths (smaller than 100 m) because of the topography influence (Jaeger et al., 2007). These statistics show that the expected anisotropy is indeed high and quite variable. Most of the data for σ_1/σ_2 and σ_2/σ_3 are in the range between 1 and 2. Fig. 1 also shows that the geostatic stress tensor most often shows anisotropy in the 3 directions (i.e. $\sigma_1 \neq \sigma_2 \neq \sigma_3$).

Gysel (1975) presents the geostatic stress tensors with respect to the tunnel alignment for two sections of the Sonnerberg tunnel, built in the Alps, in Lucerne, Switzerland. The sections are 1 km apart approximately and excavated in different types of sandstone. Table 1 shows the

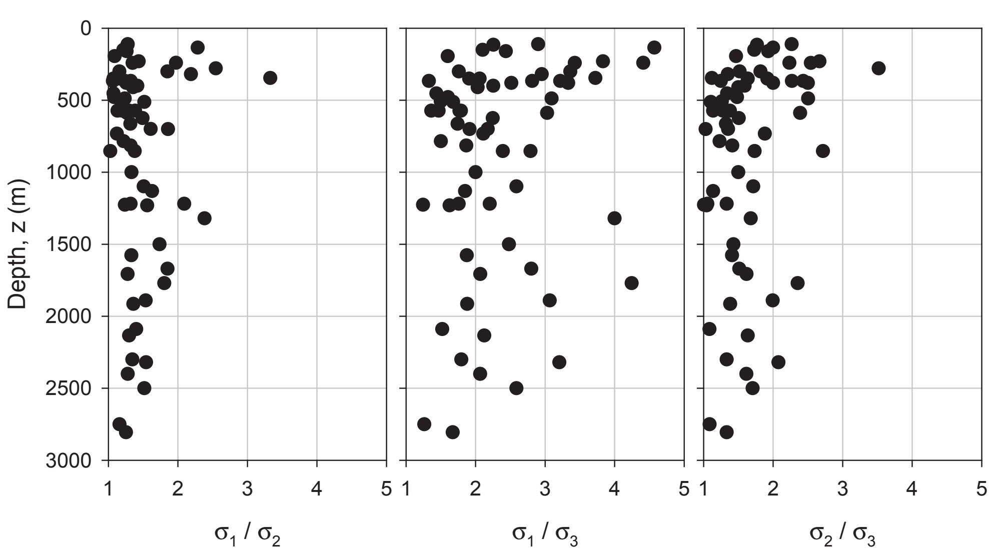

Fig. 1. Ratio of principal stresses with depth, from McGarr and Gay (1978) data compilation.