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Multivariate multiscale fractional order weighted permutation entropy of nonlinear time series

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HIGHLIGHTS

- We propose a modified multivariate weighted multiscale fractional permutation entropy.
- The complex behaviors of synthetic time series and stock markets have been successfully detected.
- Stock markets of different areas are distinguished using our method.
- The modified method allows negative information to detect deceptive cases.
- We use the idea of weighting to weaken the deception.

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ABSTRACT

In this letter, multivariate multiscale fractional permutation entropy (MMFPE) and multivariate weighted multiscale fractional permutation entropy (MWMFPE) have been proposed to provide insights for the study of time series. When measuring the dynamics of complex systems, the MMFPE and MWMFPE methods are sensitive to the signal evolution. Meanwhile, they can provide some analysis of complexity over multiple time series as well as multiple channel signals. We perform these methods on synthetic tri-variate time series to explore some of the interesting properties, especially for negative information and information deception. It can be seen that more complex system is more likely to be deceptive. The amplitude information of time series which is taken into account in the MWMFPE can weaken this deception. The methods are also employed to the closing prices and trade volume of financial stock markets from different areas. According to the MWMFPE results, the indices can be divided into three groups: (1) CAC40, HSI, NASDAQ, S&P500, (2) N225, and (3) ShenCheng, implying that it has a capacity to distinguish these financial stock market.

1. Introduction

The study of financial systems which are regulated by their own mechanism or external environment maintains a hot topic. Time series derived from financial systems are complex, and the entropy technology has been a more popular method to measure the complexity of them. Entropy analysis has gone through a great improvement and successfully applied to various fields [1–14]. Our team has done a lot of research on entropy analysis. J. Wang et al. employ multiscale entropy to traffic time series [15]. Y. Yin et al. detect the multiscale properties of financial market dynamics based on an entropic segmentation method [16]. H. Xiong et al. perform the weighted multifractal cross-correlation analysis based on Shannon entropy [17]. Besides, X. Mao et al. use transfer entropy to measure multivariate time series [18] etc.

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Permutation entropy was introduced by Bandt and Pompe [5], which quantifies the degree of complexity based on the appearance of ordinal patterns referring to the transformation in which values are replaced by their rank. In the view of its simplicity and fast calculation, the methods based on permutation entropy have been widely used in a variety of fields, such as life sciences [19–22], and financial markets [23,24]. There is no doubt that no model is perfect in every respect. Hence, B. Fadlallah et al. proposed a modification to generate weighted permutation entropy by incorporating amplitude information from the relative order structure [6]. The weighting system makes it possible to detect the abrupt changes in the data. Contemporarily, multiscale analysis has been proposed to make up for the wrong results due to the scale [25]. There is another multi-variance method which allows us to study multiple variables at the same time [9,18]. Combined all of them, multivariate multiscale permutation entropy (MMPE) and multivariate weighted multiscale permutation entropy (MWMPE) are employed to measure the complexity of more complex nonlinear systems. The computing procedure of these two methods is based on the classical Shannon entropy formula. In this letter, we extend it to the generalized fractional entropy, propose multivariate multiscale Fractional order permutation entropy (MMFPE) and multivariate multiscale Fractional order weighted permutation entropy (MWMFPE), and perform them to the synthetic data as well as financial time series. The properties of fractional calculus are introduced in details in Ref. [2].

As a generalization of the formula, fractional order generalized information entropy contains Shannon entropy as a special case. And the generalized fractional entropy has a parameter α for us to choose freely. The fractional order parameter α aims to depict more abundant complexity information metrics of dynamical systems. Different values of parameter express different meanings, some of them helps us to test "misleading events" of time series. The selection of parameter α does not have blowing of the fit and unfit quality. And when the parameter α is equal to 0, generalized fractional entropy is degenerate into Shannon entropy. Surely, it can provide more abundant dynamical properties of complex systems. Because of a higher sensitivity, the technology fractional calculus which is used in the generalized fractional entropy is useful to study signal evolution of the dynamics of complex systems. Meanwhile, its existent negative information allows us to detect deceptive cases of time series.

The remainder of the present paper has the following structure: in Section 2, we elaborate the methodology in quantifying the complexity of time series and propose the MMFPE and MWMFPE. Section 3 presents the simulation results, and Section 4 depicts the empirical results. Finally, we conclude our results in Section 5.

2. Methodology

2.1. Multiscale fractional order weighted permutation entropy

For a given discrete time series $\{x_i\}_{i=1}^{N}$, it should be coarse-grained first. We construct the multiple coarse-grained time series as follows: choose one scale factor s and average the data points within non-overlapping windows of length s. Then, the coarse-grained time series are composed of the following equation:

$$y_j^{(s)} = \frac{1}{s} \sum_{i=(j-1)s+1}^{js} x_i \tag{1}$$

Here, we have $1 \le j \le N/s$, and the length of coarse-grained time series is equal to the length of original time series divided by s. It is emphasized that using non-overlapping scales to see the multiscale properties can only be applicable to data with long length. The multiscale fractional order permutation entropy (MFPE) of coarse-grained time series with different scale factor s can be calculated.

The next step is that for a coarse-grained time series $\left\{y_j\right\}_{j=1}^{M}$, when choosing a embedding dimension d and time delay τ, we have its time-delay embedding representation $Y_k^{d,\tau} = \left\{y_k, y_{k+\tau}, \ldots, y_{k+(d-1)\tau}\right\}$ for $k = 1, 2, \ldots, M - (d-1)\tau$. To compute MFPE, these $T = M - (d-1)\tau$ sub-vectors are assigned a motif among all unique orderings of d different numbers, that is, $d!$ possible ones. Apply the following formula, we define the MFPE as the fractional order entropy:

$$H(d, \tau, \alpha) = \sum_{l} \left\{ -\frac{p\left(\pi_l^{d,\tau}\right)^{-\alpha}}{\Gamma(\alpha+1)} \left[lnp\left(\pi_l^{d,\tau}\right) + \psi(1) - \psi(1-\alpha) \right] \right\} p\left(\pi_l^{d,\tau}\right) \tag{2}$$

where $\Gamma(\cdot)$ and $\psi(\cdot)$ represent the gamma and digamma functions and $\left\{\pi_l^{d,\tau}\right\}_{l=1}^{d!}$ is the $d!$ distinct symbols. Note that expression (2) fails to obey some of the Khinchin axioms with exception of the case $\alpha = 0$ that leads to the classical Shannon entropy [2].

$p\left(\pi_l^{d,\tau}\right)$ is defined as

$$p\left(\pi_l^{d,\tau}\right) = \frac{\sum_{k \le T} \mathbf{1}_{u:type(u)=\pi_l}\left(Y_k^{d,\tau}\right)}{\sum_{k \le T} \mathbf{1}_{u:type(u)\in\Pi}\left(Y_k^{d,\tau}\right)} \tag{3}$$