Physica A 515 (2019) 31–37

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The impact of self-stabilization on traffic stability considering the current lattice's historic flux for two-lane freeway

Guanghan Peng [a,b,*], Hongzhuan Zhao [c,**], Xiaoqin Li [a]

[a] College of Physics and Electronics, Hunan University of Arts and Science, Changde 415000, China
[b] College of Physical Science and Technology, Guangxi Normal University, Guilin 541004, China
[c] College of Architecture and Transportation Engineering, Guilin University of Electronic Technology, Guilin, 541004, China

HIGHLIGHTS

- A new lattice model is proposed with self-stabilization effect for two lanes.
- Linear stability condition is obtained with self-stabilization effect on two-lane highway.
- Simulation tests verify that traffic jams are suppressed efficiently with self-stabilization effect besides lane changing.

ARTICLE INFO

Article history:
Received 21 May 2018
Received in revised form 3 August 2018
Available online xxxx

Keywords:
Traffic flow
Lattice hydrodynamic model
Self-stabilization effect
Nonlinear analysis

ABSTRACT

Self-stabilization effect reflects the adaptive change of the current lattice in traffic flow. To improve traffic flow, a new self-stabilization term in this paper is inserted into lattice hydrodynamic model for two-lane freeway. It is shown that the self-stabilization effect can increase traffic stability on two lanes whether lane changing occurs or not according to linear stability analysis. In view of numerical simulation, the self-stabilization effect enhances the stability of the traffic flow in the modified lattice hydrodynamic model for two-lane freeway.

1. Introduction

With the rapid growth of traffic flux, traffic problems are attracting more and more people's attention. In order to study traffic characteristics, many scholars set up many mathematical models [1–11]. Among them, traffic flow lattice hydrodynamic model, which firstly proposed by Nagatani [12,13], has been continuously concerned and developed in recent years. Recently, Tian et al. [14] proposed a lattice model by considering flow difference effect. Tian et al. [15] presented a lattice model with the consideration of optimal current difference effect. Gupta et al. [16,17] took into account some traffic factors such as driver's anticipation and interruption probability with passing to develop some lattice models. Ge et al. [18] adopted control method to investigate lattice model. Zhang et al. [19] further studied the lattice's self-anticipative density effect on traffic stability. Recently, Zhang [20] put forward a different lattice model with the self-stabilization effect of lattice's historical flow, which showed that the self-stabilization effect can increase the traffic stability on single lane. But above lattice models did not investigate the traffic phenomenon under lane changing. Consequently, Nagatani [21] considered lane changing behaviors to develop a two-lane lattice model. Subsequently, some factors such as driver's lane-changing aggressiveness [22], global average flux [23], average density difference [24], timid and aggressive behaviors [25] and density

* Corresponding author at: College of Physics and Electronics, Hunan University of Arts and Science, Changde 415000, China.
** Corresponding author.
E-mail addresses: pengguanghan@163.com (G.H. Peng), zhz19850108@126.com (H.Z. Zhao).

https://doi.org/10.1016/j.physa.2018.09.173

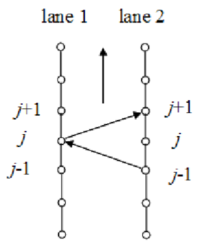

Fig. 1. The schematic model of traffic flow on a two-lane highway.

difference [26] have been investigated in lattice model under lane changing situation. However, the self-stabilization effect resulted from the current lattice's historic flux has not been taken into account in two-lane lattice models. In order to explore the influence of self-stabilization about the current lattice's historic flux on two lanes, we propose a new two-lane lattice model by considering self-stabilization effect and lane changing in the following section.

2. Modeling with the self-stabilization effect

Fig. 1 can describe the schematic diagram for two-lane highway [21]. When $\rho_{2,j-1} > \rho_{1,j}$, the lane changing rate is $\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{2,j-1}-\rho_{1,j})$. When $\rho_{2,j+1} < \rho_{1,j}$, the lane changing rate is $\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{1,j}-\rho_{2,j+1})$. Here γ means the rate constant coefficient with dimensionless. Therefore, the continuity equations [21] were written with lane changing as follows:

$$\partial_t\rho_{1,j}+\rho_0(\rho_{1,j}v_{1,j}-\rho_{1,j-1}v_{1,j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{2,j+1}-2\rho_{1,j}+\rho_{2,j-1}) \tag{1}$$

$$\partial_t\rho_{2,j}+\rho_0(\rho_{2,j}v_{2,j}-\rho_{2,j-1}v_{2,j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{1,j+1}-2\rho_{2,j}+\rho_{1,j-1}) \tag{2}$$

By adding Eqs. (1) and (2), we get the following continuity equation:

$$\partial_t\rho_j+\rho_0(\rho_jv_j-\rho_{j-1}v_{j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{j+1}-2\rho_j+\rho_{j-1}) \tag{3}$$

Here $\rho_j=(\rho_{1,j}+\rho_{2,j})/2$ and $\rho_jv_j=(\rho_{1,j}v_{1,j}+\rho_{2,j}v_{2,j})/2$. Besides, the evolution equation was adopted [21] as below:

$$\partial_t(\rho_jv_j)=a[\rho_0V(\rho_{j+1})-\rho_jv_j] \tag{4}$$

where $a=1/\tau$ shows driver's sensitivity. The optimal velocity function $V(\rho)$ is described as [21]:

$$V(\rho)=(v_{\max}/2)[\tanh(1/\rho-1/\rho_c)+\tanh(1/\rho_c)] \tag{5}$$

where ρ_c means the safety density. Moreover, based on the Nagatani's lattice model of two-lane traffic [21], some extended models [22–26] have been developed under different traffic factors. However, the impact of self-stabilization on traffic stability resulted from the current lattice's historic flux has not been investigated in two-lane lattice model. Accordingly, we think about the self-stabilization term into the evolution equation as below:

$$\partial_t(\rho_jv_j)=a[\rho_0V(\rho_{j+1})-\rho_jv_j]+\lambda a[\rho_jv_j-\rho_j(t-\tau_0)v_j(t-\tau_0)] \tag{6}$$

where $[\rho_jv_j-\rho_j(t-\tau_0)v_j(t-\tau_0)]$ means the self-stabilization effect resulted from flow difference with the information of current lattice's historical flow. τ_0 shows the historical time and λ represents the reaction coefficient. In virtue of eliminating the velocity in Eqs. (3) and (6), we certainly receive the density evolution as below:

$$\begin{aligned}&\partial_t^2\rho_j+a\rho_0^2[V(\rho_{j+1})-V(\rho_j)]-a\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{j+1}-2\rho_j+\rho_{j-1})\\&+a(1-\lambda)\partial_t\rho_j+a\lambda\partial_t\rho_j(t-\tau_0)-\gamma\left|\rho_0^2V'(\rho_0)\right|(\partial_t\rho_{j+1}-2\partial_t\rho_j+\partial_t\rho_{j-1})=0\end{aligned} \tag{7}$$

3. Linear stability analysis

To study the steady state of the uniform traffic flow, we assume the optimal velocity $V(\rho_0)$ corresponding to the constant density ρ_0 on two-lane highway. A small deviation y_j is inserted into the steady-state flow on site j as below:

$$\rho_j(t)=\rho_0+y_j(t) \tag{8}$$

Through linearizing Eq. (7), we deduce

$$\begin{aligned}&\partial_t^2y_j+a\rho_0^2V'(\rho_0)(y_{j+1}-y_j)-a\gamma\left|\rho_0^2V'(\rho_0)\right|(y_{j+1}-2y_j+y_{j-1})\\&+a(1-\lambda)\partial_ty_j-a\lambda\partial_ty_j(t-\tau_0)-\gamma\left|\rho_0^2V'(\rho_0)\right|(\partial_ty_{j+1}-2\partial_ty_j+\partial_ty_{j-1})=0\end{aligned} \tag{9}$$