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Nonlinear analysis of a new lattice hydrodynamic model with the consideration of honk effect on flux for two-lane highway

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HIGHLIGHTS

- A lattice model for two lanes is proposed under honk environment.
- Linear stability analysis is obtained.
- Nonlinear analysis is investigated about honk effect.
- Early time and long time influence of the honk on traffic flux is investigated via numerical simulation.

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ABSTRACT

Influence of the honk on traffic flux is taken to construct a new lattice model for two-lane freeway. The linear stability analysis is applied to obtain the stability condition involving the honk effect on traffic flux in two-lane system. Nonlinear analysis is implemented to get the mKdV equation for the phase transition of two-lane lattice traffic flow. Numerical simulation is worked out to authenticate that the impact of honk on traffic flux strengthens the stability of traffic flow and suppresses the traffic congestion effectively under lane changing in two-lane lattice traffic flow, which is accordance with the analytic results.

1. Introduction

With the rapid increase of traffic flux, traffic situation deteriorates more and more in modern city traffic in recent years. To explore the intrinsic mechanism of traffic jam, various kinds of traffic models have been created including macroscopic and microscopic models [1–42] by a considerable number of scholars. However, these models hardly took into account the impacts of the honk effect on the drivers' behavior. In real traffic, at the traffic bottleneck, the following vehicle often honks its horn when the preceding vehicle impedes its following vehicle from running at its current velocity in some countries such as China. In case the preceding driver receives the horn, he/her extremely probably changes lane or accelerates in view of the traffic circumstances at that time. Recently, the honk effect was investigated in a new cellular automaton model for two-lane highway [43]. Subsequently, the impacts of the honk effect on traffic behaviors have been explored in some car-following models and continuum models [44–47]. The analytical and numerical results verified that the honk effect played a positive effect on traffic dynamics. Very recently, the honk effect on the local velocity was observed in lattice model [48]. However, in real traffic, the honk effect plays more roles on the traffic flux than velocity in lattice model which is considered as macro

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traffic flow model. Up to now, the honk effect on the traffic flux has not been revealed in lattice models on two-lane highway. Thereafter, the influence of honk on the traffic flux will be considered to construct a new lattice model for two lanes in the course of modeling. The linear stability analysis and nonlinear analysis will be applied to examine the impacts of the honk effect on the traffic stability and waves. Numerical simulation will intuitively reproduce the traffic characteristic affected from the honk effect.

2. The extended model with honk effect in two-lane system

In 1998, Nagatani [41,42] firstly proposed a lattice model of single lane to study the collective dynamical evolution of traffic congestion. Inspired by this idea, many extended lattice models [49–61] have been presented by considering different traffic factors. Moreover, lane changing was taken into account to develop a two-lane lattice model by Nagatani [62] (see Fig. 1). In Nagatani's two-lane lattice model, once the density at site j − 1 (or j) on the second (or first) lane is higher than that of site j (or j + 1) on the first (or second) lane, lane changing rate from the second (or first) lane to the first (or second) lane was assumed as $\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{2,j-1}-\rho_{1,j})$(or $\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{1,j}-\rho_{2,j+1})$). Here γ represented the rate constant coefficient with dimensionless. Therefore, Nagatani [62] derived the continuity equations for two-lane highway as follows:

$$\partial_t\rho_{1,j}+\rho_0(\rho_{1,j}v_{1,j}-\rho_{1,j-1}v_{1,j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{2,j+1}-2\rho_{1,j}+\rho_{2,j-1}) \tag{1}$$

$$\partial_t\rho_{2,j}+\rho_0(\rho_{2,j}v_{2,j}-\rho_{2,j-1}v_{2,j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{1,j+1}-2\rho_{2,j}+\rho_{1,j-1}) \tag{2}$$

By merging Eqs. (1) and (2), the continuity equation with lane changing rate was obtained:

$$\partial_t\rho_j+\rho_0(\rho_jv_j-\rho_{j-1}v_{j-1})=\gamma\left|\rho_0^2V'(\rho_0)\right|(\rho_{j+1}-2\rho_j+\rho_{j-1}) \tag{3}$$

where ρ_0, ρ_j and v_j express the average density, the local density and local velocity, respectively. $\rho_j=(\rho_{1,j}+\rho_{2,j})/2$ and $\rho_jv_j=(\rho_{1,j}v_{1,j}+\rho_{2,j}v_{2,j})/2$. Moreover, the evolution equation for the two-lane highway [62] was taken as

$$\partial_t(\rho_jv_j)=a[\rho_0V(\rho_{j+1})-\rho_jv_j] \tag{4}$$

where $a=1/\tau$ is the sensitivity of a driver. $V(\rho)$indicates the optimal velocity function adopted as [62]:

$$V(\rho)=(v_{\max}/2)[\tanh(2/\rho_0-\rho/\rho_0^2-1/\rho_c)+\tanh(1/\rho_c)] \tag{5}$$

where $\rho_c$means the safety density. Recently, based on the Nagatani's lattice model, a few new lattice models were developed for two lanes by considering various traffic factors [63–74]. However, no existing lattice models mentioned the honk effect on traffic flux in two-lane system. In real traffic, traffic flux evolution is apt to maximum flux based on the current flux after front driver hears the honk effect, which shows that the honk effect will be decided by the following lattice's flux. Therefore, we proposed a new evolution equation with the honk effect on flux term as below:

$$\partial_t(\rho_jv_j)=a[\rho_0V(\rho_{j+1})-\rho_jv_j]+\mu\frac{Q_m-\rho_jv_j}{\tau'} \tag{6}$$

where μ is the reaction coefficient of honk effect term $(Q_m-\rho_jv_j)/\tau'$ and τ' is the reaction time adjusting the nth car's acceleration resulted from the honk effect. Eq. (6) means that the honk effect will be affected by the difference between the maximum velocity and current velocity. In fact, the honk effect increases the anticipation which can be described by the difference between the maximum flux and current flux. Then, one eliminates the velocity in Eqs. (3) and (6) to obtain

$$\begin{aligned}&\rho_j(t+2\tau)-\rho_j(t+\tau)+\tau\rho_0^2[V(\rho_{j+1})-V(\rho_j)]-\tau\frac{\mu}{\tau'}[-\rho_j(t+\tau)+\rho_j(t)]\\&-\tau\gamma\left|\rho_0^2V'(\rho_0)\right|[\rho_{j+1}(t+\tau)-2\rho_j(t+\tau)+\rho_{j-1}(t+\tau)]=0\end{aligned} \tag{7}$$

3. Linear stability analysis

There exists the steady state with a constant density ρ_0 corresponding to the optimal velocity $V(\rho_0)$ for the uniform traffic flow in two-lane system. Then, the solution of the homogeneous traffic flow is given by

$$\rho_j(t)=\rho_0+y_j(t) \tag{8}$$

where y_j represents a small deviation from the steady-state flow on site j. Thereafter, by substituting Eq. (8) into Eq. (7) and linearizing it, we get

$$\begin{aligned}&y_j(t+2\tau)-y_j(t+\tau)+\tau\rho_o^2V'(\rho_0)\Delta y_j(t)-\tau\frac{\mu}{\tau'}[-y_j(t+\tau)+y_j(t)]\\&-\tau\gamma\left|\rho_0^2V'(\rho_0)\right|[y_{j+1}(t+\tau)-2y_j(t+\tau)+y_{j-1}(t+\tau)]=0\end{aligned} \tag{9}$$

where $\Delta y_j=y_{j+1}-y_j$, $V'(\rho_0)=dV(\rho)/d\rho\left.\right|_{\rho=\rho_0}$. With $y_j=A\exp(ikj+zt)$ being expanded, one obtains the equation of z as below:

$$[e^{z\tau}-1][e^{z\tau}+\tau\frac{\mu}{\tau'}]+\tau\rho_o^2V'(\rho_0)(e^{ik}-1)-\tau\gamma\left|\rho_o^2V'(\rho_0)\right|(e^{ik+\tau z}-2e^{\tau z}+e^{-ik+\tau z})=0 \tag{10}$$